Quadrature Class

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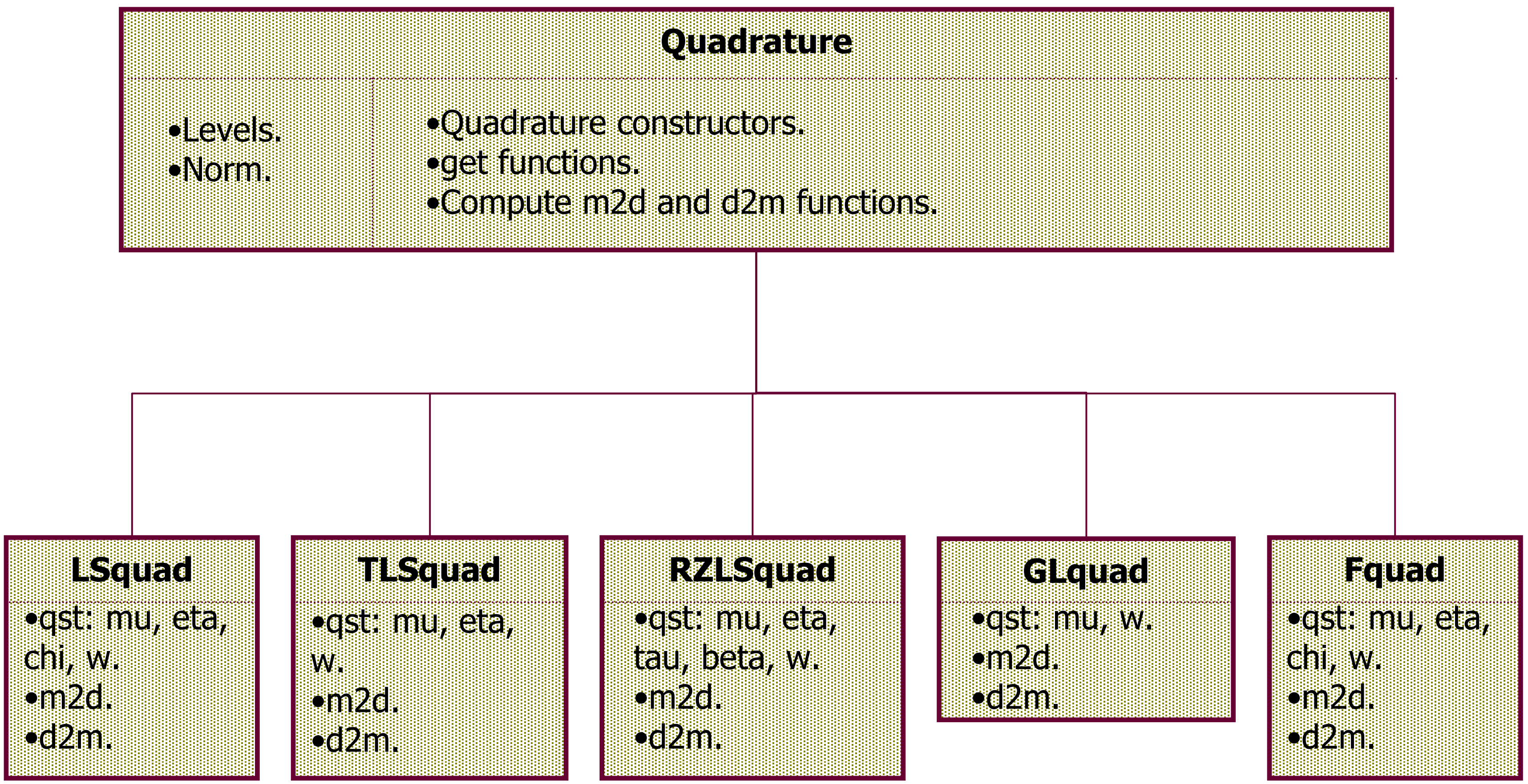
We have group angular discretization quadratures into a single class.

- **Primarily, we store quadrature points, weights and norms.**
  - ⇒ ***Struct contains each quadrature point and weight.***
  - ⇒ ***Vector of struct groups information for all directions.***
- **We store the expansion coefficient for the construction of flux moments based on angular flux information (discrete to moment) and vice versa (moment to discrete).**
  - ⇒ ***Two vector of vectors contain all coefficients for each direction: d2m and m2d.***
  - ⇒ ***Weights are build using only the real components of the spherical harmonic functions.***
- **This m2d and d2m approach is beneficial in two ways:**
  - ⇒ ***m2d and d2m are only constructed once.***
  - ⇒ ***We can expand easily to Garlekin Quadratures.***

We organize the quadrature class hierarchy based on the problem’s geometry.



We construct the quadrature by storing magnitudes and exploiting symmetries.

- **We store the magnitudes used in each quadrature point for quadratures with 2 to 24 levels.**
- **We assign the quadrature points and weights to the 1st octant.**
- **We expand to each octant using the assumed symmetries.**
- **There are three special cases:**
  - ⇒ ***RZ Level symmetric quadrature, assigns the quadrature points and calculates the angle differencing coefficients in order.***
  - ⇒ ***Gauss-Legendre quadrature calculates the quadrature points and weights.***
  - ⇒ ***Filtering quadrature always uses 6 points lying on the cartesian axis.***

We construct the m2d and d2m vectors using spherical harmonic expansions.

- **Based on Kevin Clarno's work on anisotropic scattering.**

$$\int_{4\pi} d\Omega \Sigma_s\left(\underline{r}, E' \to E, \underline{\Omega}' \bullet \underline{\Omega}\right)\psi\left(\underline{r}, E', \underline{\Omega}'\right) = \left\{ \begin{array}{l} \int\limits_{4\pi} d\Omega' \left[\sum\limits_{k=0}^{K} \frac{2k+1}{norm} \Sigma_{s,k}\left(\underline{r}, E' \to E\right) P_k\left(\mu_0\right) \right] \\ \left[\sum\limits_{l=0}^{K} \frac{2l+1}{norm} \sum\limits_{m=-l}^{l} \phi_{lm}\left(\underline{r}, E'\right) Y_{lm}\left(\underline{\Omega}'\right) \right] \end{array} \right\}$$

- **After applying the addition theorem :**

$$S_{\text{scatt}}\left(\underline{r}, E, \underline{\Omega}\right) = \sum_{k=0}^{K} \frac{2k+1}{norm} \Sigma_{s,k}\left(\underline{r}, E' \to E\right) \sum_{n=-k}^{k} \phi_{kn}\left(\underline{r}, E'\right) Y_{kn}\left(\underline{\Omega}\right)$$

$$S_{\text{scatt}}\left(\underline{r}, E, \underline{\Omega}\right) = \sum_{k=0}^{K} \frac{2k+1}{norm} \Sigma_{s,k}\left(\underline{r}, E' \to E\right) \left\{ \begin{array}{l} Y_{k0}\left(\underline{\Omega}\right)\phi_{k0}\left(\underline{r}, E'\right) + \\ \sum\limits_{n=1}^{k} \left[\begin{array}{l} \phi_{kn}\left(\underline{r}, E'\right) Y_{kn}\left(\underline{\Omega}\right) + \\ \phi_{k-n}\left(\underline{r}, E'\right) Y_{k-n}\left(\underline{\Omega}\right) \end{array} \right] \end{array} \right\}$$

- **Based on the spherical harmonic definitions:**

$$\left. \begin{array}{r} Y_{k,-n}\left(\underline{\Omega}\right) = \left(-1\right)^n Y_{kn}^{*}\left(\underline{\Omega}\right) \\ \phi_{k,-n}\left(\underline{r}, E'\right) = \left(-1\right)^n \phi_{kn}^{*}\left(\underline{r}, E'\right) \end{array} \right\} \Rightarrow S_{\text{scatt}}\left(\underline{r}, E, \underline{\Omega}\right) = \sum_{k=0}^{K} \frac{2k+1}{norm} \Sigma_{s,k}\left(\underline{r}, E' \to E\right) \left\{ \begin{array}{l} Y_{k0}\left(\underline{\Omega}\right)\phi_{k0}\left(\underline{r}, E'\right) + \\ \sum\limits_{n=1}^{k} \left[\begin{array}{l} \phi_{kn}\left(\underline{r}, E'\right) Y_{kn}\left(\underline{\Omega}\right) + \\ \phi_{kn}^{*}\left(\underline{r}, E'\right) Y_{kn}^{*}\left(\underline{\Omega}\right) \end{array} \right] \end{array} \right\}$$

The spherical harmonics functions can be expressed solely with real numbers.

$$f+f^*=2\,\mathrm{Re}[f]\Rightarrow S_{\text{scatt}}(\underline{r},E,\underline{\Omega})=\sum_{k=0}^{K}\frac{2k+1}{norm}\Sigma_{s,k}(\underline{r},E'\to E)\left\{\begin{array}{l}Y_{k0}^{\mathrm{Re}}(\underline{\Omega})\,\phi_{k0}^{\mathrm{Re}}(\underline{r},E')\\+2\sum_{n=1}^{k}\mathrm{Re}\left[\phi_{kn}(\underline{r},E')Y_{kn}(\underline{\Omega})\right]\end{array}\right\}$$

- **Rearranging:**

$$S_{\text{scatt}}(\underline{r},E,\underline{\Omega})=\sum_{k=0}^{K}\frac{2k+1}{norm}\Sigma_{s,k}(\underline{r},E'\to E)\left\{\begin{array}{l}Y_{k0}^{\mathrm{Re}}(\underline{\Omega})\,\phi_{k0}^{\mathrm{Re}}(\underline{r},E')\\+2\sum_{n=1}^{k}\left[\begin{array}{l}\mathrm{Re}\left[\phi_{kn}(\underline{r},E')\right]\mathrm{Re}\left[Y_{kn}(\underline{\Omega})\right]\\-\mathrm{Im}\left[\phi_{kn}(\underline{r},E')\right]\mathrm{Im}\left[Y_{kn}(\underline{\Omega})\right]\end{array}\right]\end{array}\right\}$$

- **We define the variables:**

$$R_{kn}=\begin{cases}\sqrt{2-\delta_{n0}}\,\mathrm{Re}[Y_{kn}]\,\forall n\geq 0\\ \sqrt{2}\,\mathrm{Im}\left[Y_{k,n}\right]\forall n<0\end{cases},\tilde{\phi}_{kn}(\underline{r},E')=\int_{4\pi}d\Omega'\left[R_{kn}(\Omega')\psi(\underline{r},E',\Omega')\right]$$

- **Applying these definition to the discrete case, d2m and m2d matrices become:**

$$S_{\text{scatt}}(\underline{r},E,\underline{\Omega})=\sum_{k=0}^{K}\sum_{n=-k}^{k}\left[\frac{2k+1}{norm}R_{km}(\Omega_m)\right]\Sigma_{s,k}\left(\underline{r},E_{g'}\to E_g\right)\tilde{\phi}_{kn}\left(\underline{r},E_g\right)$$

$$\tilde{\phi}_{kn}\left(\underline{r},E_g\right)=\sum_{m=1}^{M}w_mR_{kn}(\Omega_m)\psi\left(\underline{r},E_g,\Omega_m\right)$$

More work needs to be done …

- **The test passes isotropic scattering:**
  - ⇒ ***Source iterations test.***
  - ⇒ ***GMRES.***
  - ⇒ ***TSA , Stretch and Filter.***
- **We are searching for tests for anisotropic scattering in *xyz* and *xy* geometries.**
- **We need to change our inputs:**
  - ⇒ ***Currently only scattering order is specified at the base of the XML tree input.***
  - ⇒ ***User will specify the scattering order per isotope.***
    - The maximum scattering order over all isotopes will be the scattering order.
  - ⇒ ***User will specify type of scattering expansion.***