Ex. 3.4.1-4] The running time of the algorithm is $O(n \cdot W)$, where $n$ is the number of items and $W$ is the maximum weight the thief can steal. Assuming numeric parameters to the problem are written in binary, it takes $\log W$ bits to represent $W$. So the size of the part of the input representing $W$ is $K = \log W$. But the running time depends on $W = 2^K$, which is not polynomial in $K$ (size of the input) in general.

Ex. 3.4.2-1] Show that GRAPH-ISOMORPHISM is in NP by describing a poly time algorithm to verify a candidate solution. A candidate solution is a mapping $f$ from $V_1$ (nodes of $G_1$) to $V_2$ (nodes of $G_2$). To verify, check that the mapping is one-to-one and onto (takes time $O(V_1)$). Then check, for each pair of nodes $u$ and $v$ in $V_1$, that $(u, v)$ is an edge of $G_1$ if and only if $(f(u), f(v))$ is an edge of $G_2$ (takes time $O(V_1^2)$).

Ex. 3.4.5-1] Show that the subgraph isomorphism problem (SI) is NP-complete. SI ∈ NP: A candidate solution, given input $G_1$ and $G_2$, is a subset $S$ of the nodes of $G_2$ and a mapping $f$ from the nodes of $G_1$ to $S$. Verify as in previous exercise.
known NPC
\[ \text{problem} \]
unknown NPC
\[ \text{problem} \]

**Clique vs. IS**

Given any CLIQUE input $G$ and $K$, construct in polynomial time this IS input: $G_\overline{C}$ and $G$, where $G$ is the original CLIQUE input and $G_\overline{C}$ is the clique graph with $K$ nodes. Check that $G$ has a clique of size $K$ if and only if $G_\overline{C}$ is isomorphic to a subgraph of $G$ (i.e., if and only if $G$ contains a clique of size $K$). The point is the CLIQUE is a special case of IS.

**Prof. 34-1**

a) Independent set (IS) decision problem:

Given a graph $G$ and an integer $K$, does $G$ have an independent set of size at least $K$?

Show IS is NP-complete.

i) **IS \in NP**:

Given a candidate solution, which is a subset $S$ of the nodes of the input graph $G$, check in polynomial time if $|S| \geq K$ and if there is no edge between any pair of nodes in $S$.

ii) **Clique vs. IS**:

Given an arbitrary CLIQUE input $(G, K)$, construct in polynomial time an IS input $(\overline{G}, K)$, where $\overline{G}$ is the complement graph of $G$. Since a set of nodes $C$ is a clique in $G$ if and only if $C$ is an independent set in $\overline{G}$, $G$ has a clique of size $K$ if and only if $\overline{G}$ has an independent set of size $K$. 
c) Efficient alg. to solve IS when each vertex in \( G \) has degree 2.

Then \( G \) must consist of one or more (simple) cycles. For each cycle \( K \), number the nodes in order around the cycle \( v_1^k, v_2^k, v_3^k \ldots \).

Choose the even-indexed nodes to be in the independent set.