Effective Automatic Parallelization with Polaris

William Blume, Rudolf Eigenmann, Keith Faigin, John Grout, Jay Hoeflinger, David Padua, Paul Petersen, William Pottenger, Lawrence Rauchwerger, Peng Tu and Stephen Weatherford

Center for Supercomputing Research and Development

Coordinated Science Laboratory

University of Illinois

Abstract. The Polaris project has delivered a new parallelizing compiler that overcomes severe limitations of current compilers. While available parallelizing compilers may succeed on small kernels, they often fail to extract any meaningful parallelism from large applications. In contrast, Polaris has proven to speed up real programs significantly beyond the degree achieved by the parallelization tools available on the SGI Challenge machine. The techniques implemented are interprocedural symbolic program analysis, scalar and array privatization, symbolic dependence analysis, and advanced induction and reduction recognition and elimination. We will present preliminary results of this compiler as of Fall 94 and we will describe new techniques for runtime detection of parallelism, which we will implement in the future.

1 Introduction

Supporting standard programming languages on any kind of computer system is and has been an important issue in computer science. For parallel machine architectures this issue is not only more difficult but also crucial to make these machines easy to use. Parallel machines are more intricate and demand a deeper understanding from those users who have to exploit machine features through specific, non-standard language constructs. As machine structures are evolving rapidly, these users would have to

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repeatedly learn new machine-specific features and language elements, and they would have to repeatedly rewrite a large body of existing applications. These issues make the discipline of supporting standard languages, and thus automatic program transformations, an interesting and important research area.

For many years, our research groups have been working toward the goal of making parallel computing a practical technology. Parallelizing compilers have been playing an important role in this quest. The present project has its early roots in a compiler evaluation effort of the late 80s, where we have found that despite the success on kernel benchmarks, available compilers were not very effective on large programs [EHL91, BE92]. New measurements on a representative set of real programs were made possible, thanks to the Perfect Benchmarks® effort, which were developed in a related project [BCK+89].

Based on these observations, we have hand parallelized the program suite as a major new approach to identifying effective program transformations [EHL91, EHP94]. As a result, we have found that not only can real applications be parallelized effectively, but the transformations can also be automated in a parallelizing compiler. One issue remained: we had not actually implemented these transformations and thus not delivered the final proof that parallelizing compilers can be improved dramatically.

To resolve this issue we have implemented a new parallelizing compiler, called Polaris, and have evaluated its effectiveness. Polaris takes Fortran77 programs as its input, recognizes parallelism, and outputs Fortran77 + parallel annotations for several parallel Fortran dialects. In our current experiments we make most use of Polaris’ ability to generate SGI Challenge parallel Fortran directives. Polaris output then feeds into the code generating Fortran compiler available on the target machine.

Our compiler consists of a sizable basic infrastructure for manipulating Fortran programs. The design of this infrastructure is important in that it made the prototyping of analysis and transformation passes relatively easy. Furthermore, this infrastructure is made available to interested research and industrial organizations to develop and experiment with new compiler capabilities. Because of this, we will describe its design and design rationales in some detail in Section 2. Polaris includes many analysis and transformations passes. In Section 3 we will describe the subset on them where we feel we have made the most notable improvements over state-of-the-art algorithms. Section 3.5 describes a promising new
techniques that is currently being implemented. Finally, in Section 4 we present results of the Polaris compiler.

2 Internal Organization of the Compiler

The aim in the design of Polaris’ internal organization [FHP+94] was to create an internal representation (IR) that enforced correctness, was robust and, through high-level functionality, easy to use.

Our view of the IR is that it is more than just the structure of the data within the compiler. We also view it as the operations associated with this data structure. Intelligent functionality can frequently go a long way toward replacing the need for complex data structures and is usually a more extensible approach. Thus, we have chosen to implement the data-portion of the IR in the traditional, straightforward form of an abstract syntax tree. On top of this simple structure, however, we have built layers of functionality, which allow the IR to provide higher-level operations.

We chose to implement Polaris in the object-oriented language C++ as it both allowed us structural flexibility and gave us the desired data-abstraction mechanisms. Operations built into the IR are defined such that the programmer is prevented from violating the structure or leaving it in an incorrect state at any point in a transformation. Transformations are never allowed to let the code enter a state that will no longer represent proper Fortran syntax. The system also guarantees that the control flow graph is consistent through automatic incremental updates of this information as a transformation proceeds. The automatic consistency maintenance has drastically decreased the time required to develop new optimizations within Polaris’ production system.

Additional features that have been implemented in order to make the system robust and to maintain consistency include:

- A clear understanding of who is responsible for the deallocation of an object. The owner of an object is always responsible for its destruction. In Polaris we use the convention that a pointer is used to indicate ownership, and a reference is used to indicate that the object is not owned.
- The detection and reporting of aliased structures (structure sharing is not allowed) with a run-time error message. For example, it would be an error to create a new expression and insert it into two different statements without first making a copy of the object.

- Detection of object deletion when that object is being referenced from another part of Polaris. If the deleted object is subsequently referenced, Polaris will abort with an internal consistency error. Furthermore, dangling pointers and their associated problems are avoided by the reference counting of all objects stored in collections.

- Extensive error checking throughout the system through the liberal use of assertions. Within Polaris, if any condition or system state is assumed, that assumption is specified explicitly in a \texttt{p.assert()} (short for "Polaris assertion") statement which checks the assumed condition and reports an error if the assumption is incorrect.

In our implementation, we have followed the usual object-oriented approach in that classes are used to represent the various program structures, and that their member functions are used to manipulate the structures. These include programs, program units, statements, statement lists, expressions, symbols and symbol tables as well as a complete set of support structures which includes parameterized container and iterator classes. Each class provides extensive high-level functionality for manipulating the class instances.

Much of the implementation was intuitive and straightforward. The \texttt{Program} class, for instance, is little more than a collection of \texttt{ProgramUnits}. It includes member functions for reading complete Fortran codes, displaying them, adding additional \texttt{ProgramUnits}, and merging \texttt{Programs}.

Similarly, the \texttt{ProgramUnit} class is a holder for the various data structure elements that make up a Fortran program unit such as a statement list, a symbol table, common blocks, and equivalences.

Statements are simple, non-recursive structures kept in a list. There is no notion of statement blocks. However, we have made the implementation flexible enough that member functions which simulate the existence of statement blocks can easily be implemented on top of the current \texttt{Statement} class.
The data fields declared in the base **Statement** class (and which are *inherited* by specific classes) exist in all statements) include sets of successor and predecessor flow links, sets of memory references, and an **outer** link that points to the innermost enclosing do loop. Whenever practical, we have implemented the member functions such that any modification to a statement results in the updating of affected data, in order to retain consistency.

Each derived statement class may declare additional fields. The **DoStmt**, for example, declares a **follow** field which points to its corresponding **EndDoStmt** as well as fields for the index of the loop and the initial, limit, and step expressions. Each statement class also declares a number of member functions such as a routine that returns an iterator which traverses all of the expressions contained in the statement. Along with similar member functions in the **Expression** class, this makes it easy, for instance, to traverse all the expressions in a loop body to examine expressions for dependence analysis.

Expressions and symbols are implemented in much the same way as statements in that an abstract base class declares structures common to all elements and specific classes are derived from the base. For example, the base **Expression** class includes member functions for such operations as retrieving type and rank information, simplification and structural equality comparison. Polaris has very powerful expression comparison routines, as well as pattern-matching and replacement routines. These are based on an abstract **Wildcard** class, which is derived from Expression. To perform pattern matching, one simply creates a pattern expression (an expression that may contain wildcards anywhere in the tree) and compares this pattern to an expression using the equality matching member function. These functions have proven to be powerful and general, and are used by the reduction recognition pass, for example.

The **StmtList** class also contains much functionality which is frequently used during transformation passes. The **StmtList** class is, intuitively, a list of statements. In addition, it contains an extensive variety of high-level member functions for manipulating this list, all of which include automatic updating of control flow and loop-nesting information.

To maintain complete control of consistency inside the **StmtList** class, the manipulation of statements or statement lists are restricted by checks during the execution of Polaris. For example, the block
to be processed must be entirely well-formed with regard to multi-block statements such as do loops and block-if statements. Another example is the restriction that deleting a block containing a statement that is referenced from outside of this block is illegal. It is trapped as a run-time error during Polaris’ execution. The programmer can defer this consistency management by using a temporary copy of the code section being manipulated. In this way, one can create a section of code that is inconsistent during its creation and modification but which is checked for consistency when it is incorporated into the main data structure. These safeguards create an environment where traditionally time-consuming errors are immediately recognized and correct code can be quickly created.

3 Transformsations

In the following sections we will discuss many of the techniques that have been built into the current version of Polaris.

The first is the method of interprocedural analysis used for this stage of Polaris’ development. We have chosen to implement inline expansion for several reasons: (1) it provides us with the most information possible, (2) it allows existing intraprocedural techniques to be used, (3) it allows the calling overhead of small routines to be eliminated. After we have gained more experience with the use of inline expansion in Polaris, we are planning on augmenting it with otherforms of interprocedural analysis.

The second transformation is the recognition and removal of inductions and reductions. The third technique is symbolic dependence analysis for the recognition of parallelism. The fourth transformation is scalar and array privatization. This is one of the fundamental enabling transformations. Through advanced flow-sensitive analysis we can determine when arrays or even array sections can be replicated to reduce the storage that must be shared among the processors.

All of these techniques can be considered extensions of known compilation techniques. In fact, we have put emphasis on developing them as such, so that we could leverage off of existing technology. The most notable differences, which are primarily responsible for Polaris’ performance improvements over
available compilers are the recognition of generalized induction variables and histogram reductions, the privatization of arrays and array sections, and data-dependence analysis in the presence of symbolic, nonlinear expressions. All of these techniques had to work interprocedurally. In this paper we describe an advanced subroutine inlining facility as one of the techniques that enables us to do this. The specific sections will explain Polaris’ distinguishing features in more detail.

The final transformation discussed in this paper is a run-time technique for finding and exploiting parallelism. Even with the advanced symbolic analysis techniques and transformations that we have implemented, we find that sometimes the control flow of a region or the data dependence patterns are a function of the program input data. For these cases we are developing run-time methods for the recognition and implementation of parallelism. These techniques are not currently implemented in Polaris but will include inspector/executor [SMC91] style implementations as well as implementations based on speculative execution.

3.1 Inline Expansion

The Polaris inliner is designed to provide three types of services: complete inline expansion of subprograms for analysis, selective inline expansion of subprograms for code generation, and selective modification of subprograms.

Interprocedural analysis is a requirement for effective automatic parallelization. In Polaris, we chose to use inline expansion to allow full flow-sensitive interprocedural analysis, which is especially important for privatization. A driver routine is provided in the Polaris base to perform inline expansion either completely or up to a point defined by simple heuristics.

For inline expansion, the inliner driver is passed a collection of compilation units: one is designated as the top-level program unit. The driver repeatedly expands subroutine and function calls in the top-level program unit. This allows the inliner to handle complex argument redimensioning and retyping by generating equivalences.

The first time a subprogram is to be expanded, the inliner creates a “template” object, with sub-
program variables renamed to avoid conflicts with top-level program symbols. Each individual call site is then replaced by a copy of this template, after performing site-specific transformations on this copy, such as replacing references to formal parameters with actual parameters.

In most cases, the inliner can map formal arrays directly into the corresponding actual array in the top-level program. Occasionally, a formal array must be mapped into an equivalent, linearized version of the actual array. In practice, the range test (Section 3.3) has been able to overcome the potential loss of dependence accuracy caused by linearization.

Our inlining facility was able to completely inline all programs in our test suite with few insignificant exceptions. Some constructs are not easily expressible in Fortran after inline expansion. The constructs which are not fully supported involve the need for expressing an equivalence between non-conforming formal and actual parameters. An example is the passing of a REAL actual array to a COMPLEX formal array. This is handled automatically, but requires the favorable assumption that the variables are properly aligned in memory.

### 3.2 Induction Variable Substitution and Reduction Recognition

Induction and reduction variables form *recurrences*, which inhibit the parallel execution of the enclosing loop. Each loop iteration assigns a new value to the variable based on the value assigned in the previous iteration. In data dependence terms, this forms a cycle in the dependence graph, which serializes the loop. The following section provides an overview of Polaris’ techniques to handle these idioms. More details can be found in [PE95].

#### 3.2.0.1 Reduction variables

Most often accumulate values computed in each loop iteration, typically of the form \( \text{sum} = \text{sum} + \langle \text{expression} \rangle \). Because the “+” operation is commutative and distributive, partial sums can be accumulated on parallel processors and summed at the end of the loop. Due to the limited precision of Fortran variables, this transformation may introduce some inaccuracy, and therefore in most compilers the user has the option of disabling this transformation. However, we have
not found this to be a problem for our evaluation suite.

Polaris uses a directive to flag potential reductions of the form:

$$\Lambda(\alpha_1, \alpha_2, \ldots, \alpha_n) = \Lambda(\alpha_1, \alpha_2, \ldots, \alpha_n) + \beta$$

where $\alpha_i$ and $\beta$ are expressions that do not contain references to $\Lambda$, $\Lambda$ is not referenced elsewhere in the loop, and $\alpha$ may be null (i.e., $\Lambda$ is a scalar variable). The data-dependence pass later removes the flag if it can prove independence. The backend code generator for the SGI Challenge machine produces parallel code that implements variables flagged as reductions. More than one reduction statement can occur in a loop and they may sum into different array elements in different loop iterations. We call this case a histogram reduction. Because the SGI Challenge backend cannot handle histogram reductions, Polaris performs the necessary translation into a parallel loop.

3.2.0.2 Induction variables form arithmetic and geometric progressions which can be expressed as functions of the indices of enclosing loops. A simple case is the statement $k=k+1$, which can be deleted after replacing all occurrences of $k$ with the initial value of $k$ plus the loop index\(^2\).

Current compilers are able to handle induction statements with loop invariant right-hand-sides in multiply nested “rectangular” loops. In our manual analysis of programs we have found two additional important cases: one, when induction variables appear in the increment expression of other induction variables (we will call these coupled induction variables), and two, when induction variables occur within triangular loop nests (In triangular loop nests, inner loop bounds depend on outer loop indices).

Figure 1 shows a triangular loop nest that contains an induction variable. Polaris transforms this code into the parallelizable form shown. Notice that the resulting expression contains quadratic terms of the enclosing loop variables. In section 3.3 we will show how our dependence test investigates these non-linear subscript expressions.

The Polaris induction variable substitution algorithm performs three steps in order to recognize and substitute induction variables:

\(^2\) Assuming a normalized loop
\[ X_0 = 0 \]
\[
\text{do } I = 0, M-1 \\
\hspace{1em} X = X_0 \\
\hspace{3em} \text{do } J = 0, N-1 \\
\hspace{5em} \text{do } K = 0, J-1 \\
\hspace{7em} X = X + 1 \\
\hspace{9em} A(X) = \ldots \\
\hspace{11em} \Rightarrow \quad A((I*(N**2+N)+J**2-J)/2+K+1) = \ldots \\
\hspace{13em} \text{end do} \\
\hspace{11em} \text{end do} \\
\hspace{9em} X_0 = X_0+(N**2+N)/2 \\
\hspace{7em} \text{end do} \\
\hspace{5em} \text{end do} \\
\hspace{1em} \text{end do} \\
\]

**Fig. 1.** Simplified version of loop nest OLDA/100 from TRFD, before and after induction variable substitution.

1. Find candidate induction statements by recognizing recurrence patterns of scalar variables which are incremented by either a loop-invariant expression or an expression containing other candidate induction variables. The patterns we have encountered in our programs are relatively simple, and we have implemented a straightforward recognition scheme which finds dependence relationships between induction variables and detects cycles.

2. Compute the closed form of the induction variable at the beginning of each loop iteration (and the last value at the end of the loop) as functions of the enclosing loop indices. The total increment incurred by the induction variable in a loop body is first determined, and then this expression is summed across the iteration space of the enclosing loop. If an inner loop is encountered while computing this increment, the algorithm recursively descends into the inner loop and computes its closed form of the induction variable.

3. Substitute all occurrences of the induction variables. This step is the same as in other compilers. The substituted value is the closed form expression for the induction variable at the loop header plus any increments encountered up to the point of use in the loop body.

### 3.3 Symbolic dependence analysis

Data dependence analysis is crucial to determine what statements or loops can be safely executed in parallel. Two statement instances are data dependent if they both access the same memory location and
at least one of these accesses is a write. If two statements do not have a chain of dependence relations connecting them, then they can be executed in parallel. Also, a loop can be executed in parallel, without the need for synchronization between iterations if there are no dependences between statement instances in different iterations.

There has been much research in the area of data dependence analysis. Because of this, modern day data dependence tests have become very accurate and efficient [PP93]. However, most of these tests require the loop bounds and array subscripts to be represented as a linear (affine) function of loop index variables; that is, the expressions must be in the form \( c_0 + \sum_{j=1}^{n} c_j i_j \) where \( c_j \) are integer constants and \( i_j \) are loop index variables. Expressions not of this form are called nonlinear (i.e., they have a term of the form \( n \cdot i \) where \( n \) is unknown). Techniques have been developed to transform nonlinear expressions into linear ones (e.g., constant propagation and induction variable substitution), but they are not always successful.

In our experience with the Perfect Benchmarks, such nonlinear expressions do occur in practice. In fact, four of the twelve codes (i.e., DYFESM, QCD, OCEAN, and TRFD) that we hand-parallelized would exhibit a speedup of at most two if we could not parallelize loops with nonlinear array subscripts [BE94a]. For some of these loops, nonlinear expressions occurred in the original program text. For other loops, nonlinear expressions were introduced by the compiler. The two most common compiler passes that can introduce nonlinearities into array subscript expressions are induction variable substitution and array linearization. An example of how induction variable substitution can introduce nonlinear array subscripts is shown in Figure 1. This loop nest, taken from TRFD, accounts for about 70% of the code’s sequential execution time.

### 3.3.1 Range Test

To handle such nonlinear expressions, we have developed a symbolic dependence test called the range test [BE94b]. The range test can be considered an extension of a symbolic version of Triangular Banerjee’s Inequalities test [WB87, Ban88, HP91]. In the range test, we mark a loop as parallel if we can prove that
the range of elements accessed by an iteration of that loop do not overlap with the range of elements accessed by other iterations. We determine whether these ranges overlap by comparing the minimum and maximum values of these ranges. To maximize the number of loops found parallel using the range test, we permute the visitation order of the loops in a loop nest when computing their ranges.

The computation of the minimum and maximum values of a symbolic array access expression can be quite involved. For example, the maximum value of the expression $f(i) = n \cdot i$ for any value of $i$, where $a \leq i \leq b$, can be either $n \cdot a$ or $n \cdot b$, depending upon the sign of the value of $n$. So, to compute the minimum or maximum of an expression for a variable $i$, the range test first attempts to prove that the expression is either monotonically non-decreasing or monotonically non-increasing for $i$.

The monotonicity of an expression, say $f$, is determined by computing the forward difference of the expression $(f(i+1) - f(i))$, then testing whether this expression is greater than or equal to zero, or less than or equal to zero. If $a \leq i \leq b$, then the maximum of expression $f(i)$ for any legal value of $i$ is $f(b)$ if it is monotonically non-decreasing for $i$, $f(a)$ if it is monotonically non-increasing for $i$, and undefined otherwise. The computation of the minimum is similar.

As an example, we will show how to compute the minimum and maximum values of the subscript expression of array $A$ for a fixed iteration of the outermost loop of the loop nest shown in Figure 1. Let $f(i, j, k) = (i \cdot (n^2 + n) + j^2 - j)/2 + k + 1$ be the subscript expression for array $A$. To compute the minimum and maximum values of $f$ for any legal value of the inner pair of loops, we will first determine the minimum and maximum values of $f$ for the innermost loop, which has index $k$, then determine the minimum and maximum values that the middle loop, which has index $j$, can take for these minimum and maximum values. Since the forward difference for index $k$ is positive (i.e., $f(i, j, k + 1) - f(i, j, k) = 1$), $f$ is monotonically non-decreasing for $k$. Thus, the maximum value ($a_1$) that $f$ can take for any value of $k$ is $a_1(i, j) = f(i, j, j - 1) = (i \cdot (n^2 + n) + j^2 - j)/2 + j$. Similarly, the minimum value ($b_1$) of $f$ for index $k$ is $b_1(i, j) = f(i, j, 0) = (i \cdot (n^2 + n) + j^2 - j)/2 + 1$. For the next loop, the index $j$ is monotonically non-decreasing for both $a_1$ and $b_1$, since $a_1(i, j + 1) - a_1(i, j) = j + 1 > 0$ and $b_1(i, j + 1) - b_1(i, j) = j \geq 0$. So the maximum value ($a_2$) that $f$ can take for any legal value of
indices \( j \) and \( k \) is \( a_2(i) = a_1(i, n - 1) = (i \ast (n^2 + n) + n^2 - n)/2 \) and the minimum value \( b_2 \) is 
\[ b_2(i) = b_1(i, 0) = (i \ast (n^2 + n))/2 + 1. \]

By comparing these minimum and maximum values of array accesses, we can prove that there are no 
loop-carried dependences between these accesses. For example, there cannot be a loop-carried dependence 
from \( A(f) \) to \( A(g) \) for a loop with index \( i \) if the maximum value accessed by the \( j \)th iteration of index \( i \) 
for \( f \) is less than the minimum value accessed by the \((j + 1)\)th iteration of \( i \) for \( g \), and if this minimum 
value of \( g \) is monotonically non-decreasing for \( i \). See \[BE94b\] for other tests that use these minimum and 
maximum values of \( f \) and \( g \).

Returning to the example for Figure 1, we will now apply the dependence test described above to 
prove that \( A(f) \) does not carry any dependences for the outermost loop. From the previous example, 
we know that the maximum value that \( f \) can take for any legal value of indices \( j \) and \( k \) is \( a_2(i) = 
(i \ast (n^2 + n) + n^2 - n)/2 \) and the minimum value is \( b_2(i) = (i \ast (n^2 + n))/2 + 1 \). By the definition of the 
dependence test given above, if we can prove that \( a_2(i) < b_2(i + 1) \) and \( b_2 \) is monotonically non-decreasing 
for \( i \), \( A(f) \) cannot carry any dependences for the outermost loop. Since \( b_2(i + 1) - a_2(i) = n + 1 > 0 \), \( a_2(i) \) 
must be less than \( b_2(i + 1) \). Also, \( b_2 \) is monotonically non-decreasing since \( a_2(i + 1) - a_2(i) = n^2 + n > 0 \). 
Therefore, there are no carried dependences for the outermost loop and it can be executed in parallel.
The same dependence test can be used to prove that the other loops from Figure 1 also do not carry 
dependences.

As the previous examples have shown, the range test requires the capability to compare symbolic 
expressions. For example, we needed to test whether \( j > 0 \) or \( n^2 + n > 0 \) in the previous examples. To 
provide such a capability, we have developed an algorithm called range propagation. Range propagation 
consists of two parts. One part determines symbolic lower and upper bounds, called ranges, for each 
variable at each point of the program. The other part uses these ranges to compare symbolic expressions. 
The next subsection will describe an efficient way in which these variable ranges may be computed from 
the program's control flow. Expression comparison using ranges is done by computing the sign of the 
minimum and maximum of the difference of the two expressions, using techniques similar to those
described earlier.

A more complicated example is shown in Figure 2. This loop nest accounts for 44% of OCEAN’s sequential execution time. (Interprocedural constant propagation and loop normalization were needed to transform the loop nest into the form shown.) Current data dependence tests would not be able to parallelize any of the loops in the nest because of the nonlinear term 258 * x * j. The range test can prove all three loops as parallel. However, for it to do so, it must apply its tests on a temporary permutation of the loop nest, so that the outermost loop is swapped with the middle loop. This is necessary since the middle loop has a larger stride (258 * x) than the stride of the outermost loop (129). This causes an interleaving of the range of accesses performed by two distinct iterations of the outermost loop. By swapping the middle and outermost loops, the interleaving is eliminated, allowing all three loops to be identified as parallel.

\[
\begin{align*}
\text{do } K &= 0, X-1 \\
& \quad \text{do } J = 0, Z(K) \\
& \quad \quad \text{do } I = 0, 128 \\
& \quad \quad \quad A(258*I*J + 129*K + I + 1) = \ldots \\
& \quad \quad \quad A(258*I*J + 129*K + I + 1 + 129*X) = \ldots \\
& \quad \quad \end{align*}
\]

*Fig. 2.* Simplified version of loop nest FTRVMT/109 from OCEAN

*Banerjee’s Inequalities*, have been shown to be one of the most effective data dependence tests [PKK91] for real programs [PP93]. However, like most other known tests, it makes the assumption that array indices contain only linear subscript expressions, which we have found to be a serious limitation [BE94a]. The range test subsumes Banerjee’s Inequalities and handles many of the symbolic expressions we have seen in the Perfect Benchmarks. Because of this, one can expect this new data dependence test to be very effective in practice. Our implementation of the range test in Polaris supports these claims. For the evaluation suite of codes for Polaris, described later, we have found that applying the range test alone was sufficient to identify all important loop nests of our test programs as parallel.
3.4 Scalar and Array Privatization

Although symbolic dependence analysis will allow us to prove that many references in a loop nest are independent from each other, it would not allow a significantly greater number of important loops to be parallelized without several pre-transformations of the programs. In our experience, the most important of these transformations is array privatization [TP93].

Array privatization identifies scalars and arrays that are used as temporary work spaces by a loop iteration, and allocates a local copy of those scalars and arrays for that iteration. Such variables can then be safely ignored by the data dependence test.

To prove that a variable is privatizable, every use of that variable must be dominated by a definition of the variable in the same loop iteration. Determining the dominating definition for a scalar variable is straightforward, since a scalar is an atomic object that can only be read and written as a whole. However, since an array variable is a composite object that can be partially read and written, determining whether an array assignment dominates an array use needs an elaborate analysis of the array ranges. More specifically, the array privatizer must prove that the region of array elements referenced by the use is a subset of the region of array elements defined by the dominating assignment. Symbolic analysis techniques are often required for these region comparisons, since the regions often contain symbolic expressions.

In many cases, determining whether a region in a definition dominates a region in a use can be done using local information. However, in many other cases, it requires more elaborate symbolic analysis using global information.

A simple example where such analysis is necessary for array privatization is shown in Figure 3. To parallelize the I loop, array A must be privatized. Loop J defines the region $A(1:M_P)$, while loop $K$ uses region $A(1:M*P)$. Thus, to prove that $A$ is privatizable, we only need to prove that $M_P \geq M*P$. To prove this, we need to find out how the symbolic variables are related from their global def-use relations.

In Polaris, we use a demand-driven algorithm [TP94], based on a Static Single Assignment (SSA)
\[ S_1: \quad M = \ldots \]
\[ \ldots \]
\[ S_2: \quad MP = M \ast P \]
\[ \ldots \]
\[ \text{do } I = 1, M \]
\[ \quad \text{do } J = 1, MP \]
\[ \quad \quad A(J) = \ldots \]
\[ \quad \text{end do} \]
\[ \ldots \]
\[ \text{do } K = 1, M \]
\[ \quad \text{do } L = 1, P \]
\[ \quad \quad \ldots = A(M \ast (L-1) + K) \ldots \]
\[ \quad \text{end do} \]
\[ \text{end do} \]

**Fig. 3.** Example for array privatization

representation, to obtain global information. To obtain the SSA form, program variables are renamed such that each time the variable is defined it is given a new name. Then, each time a variable is used, it is named according to which definition reaches it. In the program shown in Figure 3, each variable is assigned only once, so no renaming is necessary to obtain the SSA form. Our demand-driven algorithm proceeds backwards from use to definition. To prove that \( MP \geq M \ast P \), the algorithm starts at loop \( J \) and backward-substitutes \( MP \) with \( M \ast P \) as defined in statement \( S_2 \). Because the goal is satisfied, the algorithm stops at this point and no further replacements are performed.

A more complicated example of the need for global information is shown in Figure 4, taken from the most time-consuming loop in BDNA. Several intermediate variables need to be privatized to parallelize the outermost loop in Figure 4. They are the scalar variables \( R \), \( P \), and \( M \), and the arrays \( \text{IND} \) and \( A \). Except for array \( A \), it is easy to determine that these intermediate variables are privatizable.

To determine whether \( A \) is privatizable in loop \( I \), it is necessary to determine the range of the use of \( A \) in loop \( L \). By analyzing the subscript and the range of the loop \( L \), it is easy to determine that the range is \( \{ A(\text{IND}(1)), A(\text{IND}(2)), \ldots, A(\text{IND}(P)) \} \). The possible dominating definition for \( A \) is in loop \( J \), where \( A \) is defined for the range \( A(1:I-1) \). To prove that the definition in loop \( J \) dominates all the uses in loop
do I = 2,N
   do J = 1, I - 1
      IND(J) = 0
      A(J) = X(I,J) - Y(I,J)
      R = A(J) + W
      if (R .LT. RCUTS) IND(J) = 1
   end do
   P = 0
   do K = 1, I - 1
      if (IND(K) .NE. 0) then
         P = P + 1
         IND(P) = K
      end if
   end do
   do L = 1, P
      M = IND(L)
      X(I,L) = A(M) + Z
   end do
end do

Fig. 4. Example from BDNA

L, we need to prove that \{A(IND(1)), A(IND(2)), \ldots, A(IND(P))\} falls in the range of \(A(1:I-1)\).

Our SSA based, demand-driven, sparse evaluation algorithm works well in situations like this where it is necessary to propagate values from complicated control structures with conditional assignments and statically assigned symbolic arrays. The demand-driven analysis determines how many elements of IND are defined in loop K making use of the fact that the subscript P for the assignment to IND(P) is a monotonically increasing variable with an initial value of 1 and step of 1. Using a monotonic variable identification technique similar to induction variable identification, the algorithm determines that all the elements in \{IND(1), IND(2), \ldots, IND(L)\} are assigned in loop K.

Now that the algorithm knows the definition point for \{IND(1), IND(2), \ldots, IND(P)\}, it can substitute the loop variant terms in \{A(IND(1)), A(IND(2)), \ldots, A(IND(P))\} with their values. Each of them takes on a value of loop index K. Because the value of K falls in the range \([1:I-1]\), \{IND(1), IND(2), \ldots, IND(P)\} will also fall in the same range. Hence all the uses of A fall within the range \([1:I-1]\) and are therefore dominated by the definition \(A(1:I-1)\). Thus, the algorithm determines that the array A is privatizable
in loop I.

3.5 Framework for Run-Time Analysis

The access pattern of some programs cannot be determined at compile time, either because of limitations in the current analysis algorithms or because the access pattern is a function of the input data. For example, compilers usually conservatively assume data dependences in the presence of subscripted subscripts. Although more powerful analysis techniques could remove this limitation when the index arrays are computed using only statically-known values, nothing can be done at compile-time when the index arrays are a function of the input data. Therefore, if data dependences such as these are to be detected, the analysis must occur at run-time. Because of the overhead involved, it is very important that run-time techniques be fast as well as effective.

The technique described in this Section is currently being implemented in Polaris.

3.5.1 Detecting data dependences at run-time

Consider a do loop for which the compiler cannot statically determine the access pattern of a shared array A that is referenced in the loop. Instead of executing the loop sequentially, the compiler could decide to speculatively execute the loop as a doall and generate code to determine at run-time whether the loop was, in fact, fully parallel. If the subsequent test finds that the loop was not fully parallel, then it will be re-executed sequentially.

To do this, it is necessary to have the ability to restore the original state when re-execution is needed. One strategy is to save the values of some arrays before starting the parallel execution of the loop and restore these values if the sequential re-execution is needed. However, in our implementation, some of the values computed during the parallel execution are stored in temporary locations and then stored in permanent locations if the parallel execution was correct.

In order to implement such a strategy, we have developed a run-time technique, called the Privatizing Doall test (PD test), for detecting the presence of cross-iteration dependences in a loop [RP94]. If there
do I = 1, 8
  ...
  A(U(I)) = ...
  ...
end do

<table>
<thead>
<tr>
<th>Position in shadow arrays</th>
<th>w_A</th>
<th>m_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>A_w</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>A_r</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A_np</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 5. The PD Test.

are any such dependences, this test does not identify them; it only flags their existence. In addition, if any variables were privatized for speculative parallel execution, this test determines whether those variables were, in fact, validly privatized. Our interest in identifying fully parallel loops is motivated by the fact that they arise frequently in real programs.

3.5.2 The PD test

The PD test is applied to each shared variable referenced during the loop whose accesses cannot be analyzed at compile-time. For convenience, we discuss the test as applied to only one shared array, say \( A \). Briefly, the test traverses and marks shadow array(s) during speculative parallel execution using the access pattern of \( A \), and after loop termination, performs a final analysis to determine whether there were cross-iteration dependences between the statements referencing \( A \).

For each iteration, the first time an element of \( A \) is written during that iteration, the corresponding element in the write shadow array \( A_w \) is marked. If, during an iteration, an element in \( A \) is read, but never written, then the corresponding element in the read shadow array \( A_r \) is marked. Another shadow array \( A_{np} \) is used to flag the elements of \( A \) that cannot be privatized: an element in \( A_{np} \) is marked if the corresponding element in \( A \) is both read and written, and is read first, for any iteration.

A post-execution analysis determines whether there were any cross-iteration dependences between
statements referencing $A$ as follows. If $\text{any}(A_w(\cdot) \cap A_r(\cdot))^3$ is true, then there is at least one flow- or anti-dependence that was not removed by privatizing $A$. If $\text{any}(A_{np}(\cdot))$ is true, then $A$ is not privatizable (some element is read before being written in an iteration). The counter $w_A$ records the total number of writes done to $A_w$ by all iterations, and $m_A$ is the total number of marks in $A_r$. If $w_A \neq m_A$, then there is at least one output dependence (some element is overwritten); however, if $A$ is privatizable (i.e., if $\text{any}(A_{np}(\cdot))$ is false), then these dependences were removed by privatizing $A$. The PD test is fully parallel and requires time $O(a/p + \log p)$, where $p$ is the number of processors, and $a$ is the total number of accesses made to $A$ in the loop.

The PD test is illustrated using the loop shown in Figure 5. The access pattern is given by the subscript arrays $T$, $V$, and $U$. Since $A_w(\cdot) \land A_r(\cdot)$ and $A_{np}(\cdot)$ are zero everywhere, the loop was a doall, but only after privatizing $A$ since $w_A \neq m_A$.

3.5.3 Performance of run-time techniques

It can be shown that if the PD test passes, i.e., the loop is in fact fully parallel, then a significant portion of the ideal speedup of the loop is obtained. “ideal” means the speedup that could be achieved knowing in advance whether or not the loop is parallel. In particular, the speedups obtained range from nearly 100% of the ideal in the best case, to at least 25% of the ideal in the worst case (as derived from the parallel model). On the other hand, if the PD test fails, i.e., the loop is not fully parallel, then the sequential execution time will be increased by the time required by the failed parallelization attempt. Since the PD test is fully parallel, this slowdown is proportional to $\frac{1}{p} T_{seq}$, where $T_{seq}$ is the sequential execution time of the loop and $p$ is the number of processors. If the target architecture is a MPP with hundreds or, in the future thousands, of processors, then the worst case potential speedups reach into the hundreds, and the cost of a failed test becomes a very small fraction of sequential execution time. Thus, speculating that the loop is fully parallel has the potential to offer large gains in performance, while at the same time risking only a small increase in the sequential execution time.

---

3 any returns the “OR” of its vector operand’s elements, i.e., $\text{any}(v(1 : n)) = (v(1) \lor v(2) \lor \ldots \lor v(n))$. 
In Figure 6, we show experimental results of a Fortran implementation of the PD test by hand on loop NLFILT/300 in a subroutine of TRACK. The measurements were made on an 8-processor Alliant FX/80 machine. The access pattern of the shared array in this loop cannot be analyzed by the compiler since the array is indexed by a subscript array that is computed at run-time. In addition, this loop is parallel for only 90\% of its invocations. In the cases when the test failed, we restored state, and re-executed the loop sequentially. The speedup reported includes both the parallel and sequential instantiations. The potential slowdown reflects the increase in total execution time that would have resulted if the PD test had shown that the loop was not fully parallel: it is expressed as the ratio between \(T_{seq} + T_{pdt}\) and \(T_{seq}\), where \(T_{pdt}\) is the time required for the PD test.

Our experimental results indicate that our techniques for loops with statically unknown access patterns usually are able to extract a large fraction of the parallelism available in the original loop. The experiments have also indicated that the overhead associated with these techniques is generally small, so that significant speedups result. Furthermore, we have found that the additional memory requirements do not make these techniques impractical for the programs we have examined.

4 Evaluation of Polaris Parallelization

We place great importance on the evaluation of our work. When we began testing commercial parallelizers in the 1980s, we found that they performed well on small, synthetic loops, but when faced with
actual scientific programs, they performed poorly. We analyzed them to determine the reasons for the deficiencies and identified several improvements that could be made in them. Now, we are implementing those improvements in the Polaris compiler, and once again we must evaluate where and how we have or have not succeeded.

4.1 The Benchmark Codes

The scientific programs that we used in our previous work were the Perfect Benchmarks®, which made it natural to use them to evaluate Polaris. We also included other scientific programs in order to demonstrate that our techniques apply to programs in general. For our first evaluation efforts, we chose 6 of the 13 Perfect codes, plus two currently-in-use codes that we obtained from the National Center for Supercomputing Applications. We have determined that many of the other Perfect codes will require the use of some run-time techniques.

From the Perfect Benchmarks, we chose the programs ARC2D, BDNA, FLO52, MDG, OCEAN, and TRFD. Three of these codes (ARC2D, BDNA, and FLO52) have proven to be at least moderately parallelizable with traditional techniques. The other three (MDG, OCEAN, and TRFD) were poorly parallelized by traditional techniques. The two NCSA codes which we chose were CMHOG and CLOUD3D.

4.2 The Evaluation Metrics

Since we have focused our efforts on “finding parallelism” in this first phase of the Polaris project, we devised a simple metric which attempts to quantify how well we achieved that. The metric is called percent parallel coverage. During the sequential execution of a program, we record how much time is spent executing each loop, determine the percentage of the overall running time of the sequential program, and call that figure the percent coverage of that loop. By adding up the coverages of each parallelized loop, we obtain the percent parallel coverage for the program. This gives us a single figure that portrays the quality of the parallelization. Even so, the percent parallel coverage is only a rough predictor of the eventual speedup that might be obtained from the code.
A second way of evaluating Polaris is to examine how well it does on loops ranked by running time. We ordered the loops in a program by average sequential running time per invocation, then divided the list into the longest-running 10%, the longest-running 50%, and finally all loops contributing at least 0.01% of the sequential running time. We tallied how many loops in each category required only “traditional” parallelization techniques, and how many required new Polaris techniques. The techniques found in the compilers we evaluated were scalar privatization, scalar reductions, recognition of induction variables in rectangular loop nests, and a simple subscript test. The major Polaris techniques were array privatization, array reductions, multi-site reductions, triangular inductions, the range test, and interprocedural analysis (as implemented through inline expansion).

4.3 The Results

Table 1 shows the resulting overall percent parallel coverage produced by the Polaris techniques for each of the eight benchmark codes, and what coverage could be gained by traditional techniques. In all cases, Polaris improved on the traditional techniques, and sometimes, quite dramatically, such as in the case of TRFD, where traditional techniques cover less than 1 percent, while Polaris found parallelism covering 99 percent of the sequential time.

We counted the number of loops in each category which were “intrinsically” serial (i.e., these could not be hand parallelized), how many required at least one of Polaris’ techniques to be parallelized, how many could be parallelized by traditional techniques, and the total number of loops. These results were catalogued for the longest-running 10%, 50% and all loops (as described above). The results are displayed for each program in Table 2.

This table shows that in all percent brackets, Polaris’ techniques are able to parallelize significantly more loops than traditional techniques. The fact that this ratio is most pronounced in the top 10% loops demonstrates the real impact of the new technology. Quite often we found that Polaris’ techniques could parallelize the outer loop in a nest, while traditional techniques were sufficient only for the inner loops. In our most recent experiments we have been able to confirm these improvements with actual speedup.
comparisons on an SGI Challenge machine. In all programs of our test suite where the commercially available parallelizer failed to achieve performance gains, Polaris demonstrated significant speedups.

5 Conclusion

We have presented Polaris, a new parallelizing compiler, developed at the University of Illinois. Polaris includes a powerful basic infrastructure for manipulating Fortran programs and a number of improved analysis and transformation passes, notably subroutine inline expansion, symbolic analysis, induction and reduction variable recognition, data-dependence analysis, array privatization, and run-time analysis.

The current prototype of the Polaris compiler is able to parallelize our first evaluation program suite significantly better than available compilers. In many cases this is as good as the best manual parallelization.

Previous attempts at automatic parallelization were only successful for two of the programs in our test suite. Now we are successful in half of the Perfect Benchmarks and we expect to further increase this portion in the near future. This is a substantial improvement which expands the set of programs for which parallelizing compilers are successful from small and simple codes to medium sized and complicated. We have come a significant step closer to the goal of making parallel computing available to the broad user community.

Some remaining issues are the representativeness of our program suite and the machine model we are using. We believe that the Perfect Benchmarks plus the "NCSA suite" are a good starting point for a truly representative high-performance computer workload. The fact that our newly inspected programs have confirmed previous findings about effective parallelization techniques indicates that we are in fact converging in our search for the right compiler ingredients.

The output of Polaris is suitable for machines that provide a global address space. To use Polaris on message-passing machines one will need to develop complementing optimization techniques. We do not plan to develop these techniques since a number of such projects are currently underway. Furthermore,
global address-based features will most likely be part of many parallel machines in the near term. This is already starting to happen as can be seen in the recent MPP announcements of Cray and Convex. However, it is important to note that Polaris’ innovation is in improved recognition of parallelism, which is a necessary step for porting programs to any parallel machine available today.

References


* CSRD reports are available via anonymous FTP from ftp.csrd.uiuc.edu:CSRD,Info, or the World Wide Web site http://www.csrd.uiuc.edu


<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>Polaris</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDG</td>
<td>99.95</td>
<td>68.54</td>
</tr>
<tr>
<td>ARC2D</td>
<td>99.94</td>
<td>69.96</td>
</tr>
<tr>
<td>TRFD</td>
<td>99.90</td>
<td>&lt;0.01</td>
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<td>FLO52</td>
<td>99.79</td>
<td>91.06</td>
</tr>
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<td>BDNA</td>
<td>97.95</td>
<td>32.33</td>
</tr>
<tr>
<td>OCEAN</td>
<td>95.42</td>
<td>32.58</td>
</tr>
<tr>
<td>CLOUD3D</td>
<td>84.55</td>
<td>43.09</td>
</tr>
<tr>
<td>CMHOG</td>
<td>81.93</td>
<td>17.11</td>
</tr>
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</table>

**Table 1.** Percent coverage of the serial execution time of loops that can be parallelized with Polaris techniques and traditional techniques.

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>top 10%</th>
<th>top 50%</th>
<th>all loops</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>serial</td>
<td>total</td>
<td>serial</td>
</tr>
<tr>
<td></td>
<td>Pol/trad</td>
<td></td>
<td>Pol/trad</td>
</tr>
<tr>
<td>AR2D</td>
<td>1/3</td>
<td>8</td>
<td>8/32/28</td>
</tr>
<tr>
<td>FLO52</td>
<td>2/4</td>
<td>6</td>
<td>4/23/20</td>
</tr>
<tr>
<td>TRFD</td>
<td>1/2</td>
<td>3</td>
<td>3/10/3</td>
</tr>
<tr>
<td>MDG</td>
<td>2/3</td>
<td>5</td>
<td>3/14/8</td>
</tr>
<tr>
<td>BDNA</td>
<td>0/5</td>
<td>6</td>
<td>0/27/5</td>
</tr>
<tr>
<td>OCEAN</td>
<td>0/5</td>
<td>6</td>
<td>0/27/5</td>
</tr>
<tr>
<td>CLOUD3D</td>
<td>8/12</td>
<td>14</td>
<td>14/51/32</td>
</tr>
<tr>
<td>CMHOG</td>
<td>1/4</td>
<td>5</td>
<td>2/21/14</td>
</tr>
</tbody>
</table>

**Table 2.** Loop count in the categories: serial, needing Polaris/traditional techniques, total loops, for the evaluation codes. The loops were ordered by their average serial execution time, then divided into the groups: top 10%, top 50%, and all loops with 0.01% or more of the total serial time of the program.