The Value Evolution Graph and its Use in Memory Reference Analysis *

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Abstract

We introduce a framework for the analysis of memory reference sets addressed by induction variables without closed forms. This framework relies on a new data structure, the value evolution graph (VEG), which models the global flow of values taken by induction variable with and without closed forms. We describe the application of our framework to array data-flow analysis, privatization, and dependence analysis. This results in the automatic parallelization of loops that contain arrays addressed by induction variables without closed forms. We implemented this framework in the Polaris research compiler. We present experimental results on a set of codes from the PERFECT, SPEC, and NCSA benchmark suites.

1. Motivation

The analysis of memory reference sets is crucial to important optimization techniques such as automatic parallelization and locality enhancement. This analysis gives information about data dependences within or across iterations of loops, about potential aliasing of variable names and, most importantly about the flow of values stored in program memory. The analysis of memory references reduces to an analysis of the addresses used by the program, or, more specifically in the case of Fortran programs, an analysis of the arrays indices. A large body of research has been devoted to this field yielding significant achievements. When index functions are relatively simple expressions of the loop induction variables and the array references are not masked by a complex control flow, then the analysis is relatively straight forward. For example, if in a loop an array is indexed through an affine function of the loop induction variable and the references are control flow insensitive then the data dependence analysis can be performed accurately and, if possible and profitable, the loop can be parallelized.

Unfortunately, arrays are not always referenced in such a simple manner. Sometimes the values of the addresses used are not known during compilation, e.g., when the values of the addresses are read from an input file or computed within the program (use of indirection arrays). In other situations although the addresses are expressed as a simple function of the loop induction variable, the control flow that masks the actual references makes it impossible to compute a closed form of the index variable and thus very difficult to perform any meaningful analysis.

Some of these difficulties have been addressed in the recent past by using run-time analysis and speculative optimizations for loops that cannot be analyzed statically, e.g., loops with input dependent reference patterns [22].

Recently, Hybrid Analysis [25] has improved the accuracy and performance of optimizations by bridging the gap between static and run-time analysis. In this paradigm, the partial results of static, compile-time analysis can be saved and used during a dynamic analysis phase, when all statically unknown values are available.

However, despite recent progress, memory reference analysis and subsequent loop parallelization, cannot be performed with sufficient accuracy when arrays are indexed by subscripts that cannot be expressed as a closed form of the loop induction variable. Arrays cannot be proved independent because their indeces cannot be analyzed with classical data dependence techniques and indeces of arrays (addresses) cannot be computed independently by each iteration (or processor). In this paper we propose the Value Evolution Graph (VEG) as a novel representation for the value flow of induction variables that cannot be expressed as a simple algebraic function of their loop index. We show how this technique can improve the accuracy of data dependence analysis, privatization and the recognition of certain classes of memory reference patterns, such as pushback sequences. We show how these improved techniques can lead to the automatic parallelization of a larger number of codes than ever before.

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1.1. Background and a Motivating Example

Recurrences with closed forms are those in which the $i$-th term can be written as an algebraic formula of $i$. In recurrences with closed forms most relations between values are proved using symbolic calculus. For example, references to arrays using recurrences with closed forms, can be meaningfully expressed using systems of in-equations [27, 21, 14] or triplet-based notations [15, 10] containing the closed form terms and other symbolic values such as loop bounds. We will not address such recurrences in this paper. When a recurrence with no closed form is used to index an array, the corresponding memory reference set cannot be summarized using an algebraic formula. For example, the algebraic expressions for the index of array $A$ at line 8 in Fig. 1(a) for iterations $k$ and $k+1$ are identical, $p$, but their values always differ. Hence, we need to develop alternative analysis techniques that can deal with such cases.

There are various uses for information about recurrence values. In the example in Fig. 1, we can find array $A$ independent in the loop at line 3 if we show that $q < p$ and that the values of $p$ are different in any two iterations that write to $A$. We can propagate the values stored in array $A$ in the loop at line 3 to where they are used at line 13 if we know that the set of the definition indices covers the set of use indices. We can propagate the values in $B$ if we know that $p \leq 2 + \text{old}$ (the value of $p$ at statement 12).

1.2. Our Solution: The Value Evolution Graph

To solve the problems presented above we propose to model the value flow of the recurrences without closed form with the Value Evolution Graph (VEG) and use it to obtain sufficient information to allow parallelization.

The reference pattern on array $B$ in Fig. 1(a) uses a recurrence with closed form $q(i) = i$, which was substituted in Fig. 1(b). It is easy to prove that there are no loop carried dependences on $B$ because the index of $B$ is expressed as an analytical function of the loop index. However, there is no such formula for the index of $A$ because it is indexed by $p$, which is defined by a recurrence without a closed form (due to conditional incrementation). Fortunately, data dependence analysis does not require us to have closed form solutions, but rather to prove relations between the index sets corresponding to different iterations. In order to prove array $A$ independent, we first need to show that statements 6 and 8 are independent. Note that at statement 6 we read from $A$ at offsets between $[1:\text{old}]$, and at 8 we write based on all the values of the recurrence on $p$. We can do it by finding all the values of the recurrence – its image – and prove that they do not intersect $[1:\text{old}]$. We also need to prove that statement 8 does not cause cross-iteration dependences by itself. We can do it by proving that the value of the index at line 8 always takes a positive step.

The Value Evolution Graph shown at the bottom of Fig. 1(c) translates the problems of computing the step, image, and last value of the recurrence within the first loop into graph problems. Although the idea of value flow in the program is not new [24, 29, 30], the VEG offers unique features and functionality needed by various analyses (Sec. 2). We have integrated the VEG into a generic memory reference analysis framework that can solve multiple classes of optimization problems in the presence of recurrences without closed forms.

1.3. Our Contribution

We believe that this paper makes the following original contributions:

- We propose the Value Evolution Graph that can represent the data flow in recurrences used as array indices which have no closed form solutions. The graphs are pruned based on control dependence predicates and produce tighter value ranges than abstract interpretation methods.

- Unlike the previous efforts of looking for patterns in the code text, we can analyze partially aggregated and classified memory descriptors. This single generic approach both extends and unifies in a single framework...
2. The Value Evolution Graph (VEG)

Finite recurrences are usually described by an initial value, a function to compute an element based on the previous one (an evolution function), and a limiting condition. Depending on the evolution function’s formula, in certain cases we can evaluate important characteristics even for recurrences without closed forms: the distance between two consecutive elements, the image of the recurrence, i.e., the set of all values it may take, and the last element in the sequence.

We introduce the Value Evolution Graph (VEG), a compiler representation for the flow of values across arbitrarily large and complex program sections, including, but not limited to, recurrences without closed forms. Consider the loop at line 3 in Fig. 1. It performs a repeated conditional push to a stack array $A$. The stack pointer is stored in variable $p$. Due to the fact that $p$ is incremented conditionally, there is no closed form for the recurrence that defines its value. We represent values as Gated Static Single Assignment (GSA) [2] names. In GSA, there are three types of $\phi$-nodes. $\gamma$ nodes merge two values on different forward control flow paths. $\mu$ nodes merge a loop back value with a loop incoming value. $\eta$ nodes merge the outcome of a loop with the value before the loop. While this helps to discern between the values of $p$ on the left and right hand side of the assignment at line 7 respectively, it does not differentiate between the value of $p$ at line 8 in successive iterations. However, it makes it easy to determine that the stack array is written only at position $p_2$, and that $p_2$ is always the result of an addition of 1 to $p_1$. The subgraph consisting of $\{p_1, p_2, p_3\}$ (in Fig. 1(c)) represents the value flow between different GSA names for $p$ in a single iteration of the loop. Each edge label represents the value added to its source to obtain its destination. The dashed edge carries values across iterations, but is not part of the VEG as it does not contribute to the flow of values within an iteration. We can employ well-known graph algorithms to prove that the distance between two consecutive values of $p_2$ is always 1, which makes the write to $A(p_2)$ be a stack push operation.

We will show how we construct the VEG in general, and how we run queries on it to compute recurrence characteristics over complex program constructs, such as loop nests, complex control flow, and subprogram calls.

2.1. Formal Definition

We define a value scope to be either a loop body (without inner loops), or a whole subprogram (without any loops). Immediately inner loops and call sites are seen as simple statements. We treat arrays as scalars and assume that programs have been restructured such that control dependence graph contains no cycles other than self-loops at loop headers. We have implemented such a restructuring pass in our research compiler.

Definition. Given a value scope, the Value Evolution Graph is defined as a directed acyclic graph in which the nodes are all the GSA names defined in the value scope and the edges represent the flow of values between the nodes.

Nodes. In addition to the nodes defined in the value scope, we add, for every immediately inner loop, the set of GSA names that carry values outside the inner loop. An example is $p_1$ in Fig. 1.

Such nodes appear both in their current value scope graph as well as in the immediately outside value scope graph. They are called $\mu$ nodes in the context of the graph corresponding to the inner value scope and are displayed as double circles. Nodes representing variables assigned values defined outside their scope are called input nodes and are labeled with the assigned value (they are displayed as rectangles). The $\mu$ and input nodes are the only places where values can flow into a VEG. Values can flow out of the VEG through $\mu$ nodes only.

Edges. An edge between two variables $p$ and $q$ represents the evolution from $p$ to $q$, defined as the function $f$, where $q = f(p)$. The evolution belongs to a scope if $p$ and $q$ are defined within the scope, and all symbolic terms in $f$ are defined outside it. We represent four types of evolutions, additive and multiplicative for integer values and $or$ and $and$ for logical values. We represent an evolution by its type and the value of the free term. Certain evolutions can be com-
Table 1. Extracting evolutions from the program.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Edge</th>
<th>Ev. Type</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 = a + \text{exp} )</td>
<td>( a \rightarrow b_1 )</td>
<td>+</td>
<td>exp</td>
</tr>
<tr>
<td>( b_1 = a \cdot \text{OR} \cdot \text{exp} )</td>
<td>( a \rightarrow b_1 )</td>
<td>( \lor )</td>
<td>exp</td>
</tr>
<tr>
<td>( b_1 = a \cdot \text{AND} \cdot \text{exp} )</td>
<td>( a \rightarrow b_1 )</td>
<td>*</td>
<td>exp</td>
</tr>
<tr>
<td>( b_1 = a )</td>
<td>no edge, mark input node</td>
<td>Default</td>
<td>Identity</td>
</tr>
<tr>
<td>( b_1 = \gamma(b_0, b_1) )</td>
<td>( b_1 \rightarrow b_2 )</td>
<td>Default</td>
<td>Identity</td>
</tr>
<tr>
<td>( b_2 = \mu(b_0, b_1) )</td>
<td>no edge, mark ( \mu ) node</td>
<td>Default</td>
<td>Identity</td>
</tr>
<tr>
<td>( b_2 = \eta(b_0, b_1) )</td>
<td>( b_1 \rightarrow b_2 )</td>
<td>Default</td>
<td>Loop effect</td>
</tr>
<tr>
<td>CALL sub((b_1 \rightarrow b_2))</td>
<td>( b_1 \rightarrow b_2 )</td>
<td>Default</td>
<td>sub effect</td>
</tr>
</tbody>
</table>

posed along a path symbolically. For instance, the evolution along path \( p_1 \rightarrow p_2 \rightarrow p_3 \) is an additive evolution with value \( 1 + 0 = 1 \). Instead of keeping a single value for an evolution, we store a range of possible values. This allows us to define an aggregated evolution from a node \( p \) to a node \( q \) as the union of the evolutions along all paths from \( p \) to \( q \). For example, the aggregated evolution from \( p_1 \) to \( p_2 \) is \([0:1]\) which represents the union of the evolution \([0:0]\) along path \( p_1 \rightarrow p_3 \) and the evolution \([1:1]\) along path \( p_1 \rightarrow p_2 \rightarrow p_3 \).

**Complexity.** VEGs are as scalable as the GSA representation of the program since the number of nodes in all VEGs is at most twice the number of GSA names in the program and every node corresponding to a \( \phi \) definition has the same number of incoming edges as the number of \( \phi \) arguments. All other nodes have at most one incoming edge.

2.2. Value Evolution Graph Construction

Table 1 shows how we create edges from their corresponding statements. For now, we support only one evolution type per VEG. This evolution type is given by the first evolution we encounter, and is called the default type of the graph. If a value is computed in a way different from the ones shown in the table, we conservatively transform it into an input node and label it with \( [-\infty : +\infty] \) (or \( [.FALSE.:TRUE.] \)). If it is computed in an assignment statement, then we try to find a closer range for the right hand side of the statement. We compute the aggregated evolution of an entire recurrence as the aggregated evolution, over all iterations, from the \( \mu \) node to all nodes that may carry evolutions to the next iteration. We draw an edge from the value of the \( \mu \) node to the corresponding value on the left hand side of the corresponding \( \eta \) definition, and we label it with the aggregated evolution of the inner recurrence. Fig. 1(c) shows such an edge between \( p_1 \) (a \( \mu \) node in the inner recurrence \( \{p_1, p_2, p_3\} \)) and \( p_4 \). The range \([0:old]\) is a result of multiplying the range of the aggregated evolution from \( p_1 \) to \( p_3 \), \([0:1]\), with the iteration count of the loop, old. When values are obtained as a result of a subprogram call, we add edges to represent the aggregated value evolutions of the OUT actual arguments (and global variables) as functions of IN actual arguments (and global variables). In the last line in Table 1, \( b_2 \) and \( b_1 \) are the OUT and IN arguments respectively.

The VEGs are built in a single bottom-up traversal of the whole program. The call graphs and the loop nest graphs of each program are traversed in reverse topological order. Within each scope we identify all definitions, build edges and associate input values. We use aggregated information from inner loops and called subprograms as shown in Table 1. We compute the aggregated value evolution for all the recurrences associated with the loops using shortest/longest path algorithms that are linear in the size of the graph (number of edges + number of nodes). We compute the shortest and longest paths between every \( \mu \) and input node and every other node. If every node is reachable from exactly one \( \mu \) node and there are no input nodes, the complexity of the algorithm is linear in the number of GSA names + the number of arguments in all the \( \phi \) nodes in the program. If more than one \( \mu \) node can reach one same other node (coupled recurrences), the complexity may increase by a factor of at most the number of coupled recurrences.

2.3. Queries on Value Evolution Graphs

We obtain needed information about the values taken by induction variables by querying the VEG. All the queries we support are implemented using shortest path algorithms. Since all the VEGs are acyclic, these algorithms have linear complexity.

**Distance between two values in two consecutive iterations of a loop.** Given two GSA variables (possibly identical) and a loop, we can compute the range of possible values for the difference between the value of the second variable in some iteration \( i+l \), and the value of the first variable in iteration \( i \). For recurrences without closed forms, this computes the distance between two consecutive elements. In the example in Fig. 1, the distance between \( p_2 \) in iteration \( i \) and \( p_2 \) in iteration \( i+1 \) is exactly \( l \). This information can be used to prove that the write pattern on array \( A \) at statement \( \delta \) cannot cause any cross-iteration dependences. The value of the distance between a source node and a destination node across two consecutive iterations of a loop can be used for comparisons only if the destination node is not reachable from an input node.

**Range of a variable over an arbitrarily complex loop nest.** Given a GSA variable and a loop, we can compute the range of values that the variable may take over the iteration space of the loop. For recurrences without closed
forms, this computes its image and can be used to evaluate the last element. In the example in Fig. 1, the range for variable \( p_2 \) over the loop is \([p_0+1:p_0+\text{old}]\). This information is crucial for proving that the write pattern on array \( A \) at statement 8 cannot have cross-iteration dependences with the read pattern at statement 6 (they are contained in disjoint ranges \([p_0+1:p_0+\text{old}]\) and \([1:p_0]\) respectively). This information is computed in \(O(d)\) time, where \(d\) is the depth of the loop nest between the given loop and the definition site of the given variable.

**Global value comparison.** Given two GSA variables in the same subprogram, we can compare their values even if they are not in the same value scope, by comparing their ranges in a larger common scope. This information can be used to prove either an order between their values or their equality and which in turn can be used in many compiler analyses.

### 2.4. VEG Conditional Pruning

We can prune a VEG by removing certain edges that cannot be taken when based on the truth value of a condition. The shortest path algorithms used to compute aggregated evolutions will then produce tighter ranges. Consider the code shown in Fig. 2 (a). Because we do not know anything about the value of \( \text{cond} \), we cannot compare the values of \( p_1 \) and \( p_5 \), information that is needed to determine if the memory read at offset \( p_0 \) in array \( A \) is always covered by the write at offset \( p_1 \). Based on its corresponding VEG (Fig. 2 (c)), we can only infer that \( p_5 \in [p_1-1:p_1+1] \).

The GSA path technique [28] describes how control dependence relations can be used to disambiguate the flow of values at \( \gamma \) gates. The GSA path technique can infer that at line 11 condition \( f_3.EQ.1 \) holds true, which implies also \( f_3.GT.0 \) holds true. To the VEG, this means that value \( p_5 \) comes from \( p_4 \) and not directly from \( p_3 \). With the VEG pruned using this information (Fig. 2 (d)), we have \( p_5 \in [p_1-1:p_1] \).

We have improved on [28] by using the VEG to trace back ranges extracted from given control dependence predicates. The read from array \( A \) at line 11 is guarded by condition \( f_3.EQ.1 \). This implies \( f_3.EQ.1 \) holds true. From this predicate, we extract the range \([1:1]\) for \( f_3 \). In Fig. 2 (b), we trace this range for \( f_3 \) backward to see where it could have come from. Since the initial value for \textit{input} node \( f_1 \) is 0, and the edge \( f_1 \rightarrow f_3 \) has weight 0, the only range that can be produced on the path \( f_1 \rightarrow f_3 \) is \( 0+0=0 \). The GSA gate \( f_3=\gamma(f_1,f_2,\text{cond}) \), associates the pair \((f_1,f_3)\) with condition \( \text{NOT} \). Since \( f_3 \) cannot come from \( f_1 \), \( \text{NOT} \) must be false, thus \( \text{cond} \) must be true. The same predicate, \( \text{cond} \), controls the other gate, \( p_5=\gamma(p_1,p_2,\text{cond}) \). Since \( \text{cond} \) holds true, \( p_5 \) must have come from \( p_3 \), and not from \( p_1 \). So the edge \( p_1 \rightarrow p_5 \) cannot be taken. This leads to the graph in Fig. 2 (e). On the pruned graph in Fig. 2 (e), \( p_5 = p_1+1+0-1+0 = p_1 \), which proves the read at line 11 covered by the write at line 1.

This method improves on [28], leads to more accurate ranges than the abstract interpretation method used in [4], and can solve classes of problems that [29] cannot. One use of VEG conditional pruning is presented in Sec. 4.

### 3. VEG-based Memory Reference Analysis

#### 3.1. Memory Reference Classification

Memory Classification Analysis (MCA) [15] is a general dataflow technique used to perform data dependence tests and privatization analysis, but which is also usable in any optimization that requires dataflow information, such as constant propagation. For a given program context – a statement, loop, or subroutine body – MCA classifies all memory locations accessed within the context in \textit{Read Only} (RO), \textit{Write First} (WF) and \textit{Read Write} (RW). The RO set records all memory locations that are only read (never written); the WF set records all memory locations that are first written, then possibly read and/or written again; the RW set includes all other memory locations. We perform MCA in a single bottom-up traversal of the program by aggregating and classify memory locations across larger and larger program contexts. For instance, if a variable is RO in a statement and WF in the following statement, it is classified as RW for the block of two statements.
Do400.

Figure 3.

<table>
<thead>
<tr>
<th>PROGRAM main</th>
<th>SUB build (p0→p5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 DO k = 1, 100</td>
<td>1 old = p0</td>
</tr>
<tr>
<td>2 old = q1</td>
<td>2 DO i = 1, 100</td>
</tr>
<tr>
<td>3 CALL build (q1→q2)</td>
<td>3 IF (ten (old, p1))</td>
</tr>
<tr>
<td>4 ... = A( old: q2−1)</td>
<td>4 DO j = p1, p1+9</td>
</tr>
<tr>
<td>5 ENDO</td>
<td>5 A(j) = ...</td>
</tr>
<tr>
<td>...</td>
<td>6 ENDO</td>
</tr>
<tr>
<td>FUNCTION ten (b, c)</td>
<td>7 ELSE</td>
</tr>
<tr>
<td>1 DO j = b, c-1</td>
<td>8 A(p1) = ...</td>
</tr>
<tr>
<td>2 IF (A(j)...)</td>
<td>9 p3 = p1+1</td>
</tr>
<tr>
<td>3 RETURN . F.</td>
<td>10 ENDF</td>
</tr>
<tr>
<td>4 ENDF</td>
<td>11</td>
</tr>
<tr>
<td>5 ENDO</td>
<td>12 ENDO</td>
</tr>
<tr>
<td>6 RETURN . T.</td>
<td>13</td>
</tr>
<tr>
<td>7 END</td>
<td>13</td>
</tr>
</tbody>
</table>

Figure 4. Code extracted from EXTEND.do400.

The most important memory classification process takes place at loop level (Fig. 3). The expand() operation generates the access pattern of the whole loop based on the access pattern within an iteration. For instance, in the example in Fig. 1, the W F pattern for array B within an iteration of the loop at line 3 is \{i\}. Across the entire loop, it is \(\bigcup_{i=0}^{1} \{i\} = [1:100]\). When the recurrence has no closed form, these operations cannot be performed symbolically. However, we can use the VEG to detect contiguous sequences of memory locations indexed by recurrences without closed forms. These sequences, found in subroutine contiguous-write are used to adjust the results of expand, as shown in subroutine update.

Consider the example in Fig. 4. Conceptually, the loop in program main performs a repeated pushback on array A, based on index q. The stack array A is also read at line 4 in program main and at line 2 in function ten. Both reads are to elements that have been pushed within the same iteration of the loop in program main, thus they are covered by writes. Consequently, array A is privatizable.

Traditional analysis fails because of the conditional incrementation of the index p by either 10 or 1. Recent work [18, 30, 31] focused on statement-level pattern matching of recurrence expressions. These approaches fail to relate the write to A(j) at line 5 in subroutine build to p. Also, they cannot handle the presence of read memory references and recurrences over multiple variables (q, old, p).

Our approach is to aggregate memory references symbolically using VEG information. Our addition to MCA does not require new data structures, as the new information is used to refine the existent RO, WF, and RW descriptors. We aggregate the reference pattern over the loop at line 4 in subroutine build into [p1:p1+9]. We cannot aggregate the reference pattern over the outer loop in subroutine build because the recurrence on p1 has no closed form. Regardless of the value returned by function ten, we can see that the write pattern is contiguous, i.e. it has no gaps between any two successive iterations. The write access pattern can be aggregated across the whole loop as [p0:p5-1]. At the beginning of any iteration i, the extent of the contiguous written section in previous iterations is [p0:p5-1]. The read from A at line 2 in function ten is always within [old:p5-1]. We can prove it is covered by previous writes, since old = p0. Also, we can find that the extent of the contiguous write for the whole loop is [p0:p5-1]. At the call site in program main, this translates into [q1:q2-1], which covers the successive reads completely within every iteration. We solved this MCA problem not based on the closed form of the index but rather on the information about the recurrence exposed by the VEG.

In order to parallelize the loop in program main, we still have to prove that there are no cross-iteration output dependencies. We do it by proving that the per-iteration descriptor, [q1:q2-1], is increasing, i.e. it has no overlaps. A VEG query is used to evaluate the step from q2-1 to q1 across two successive iterations and to prove it is positive.

3.2. Memory Reference Sequence Classification

A memory reference sequence is increasing in a loop if every access index in iteration \(i + 1\) is strictly larger than any index in iterations \(1\) to \(i\) (Fig. 5(a)). It is contiguous in a
Table 2. Uses of memory reference sequence classification for the parallelization the outer loop of a doubly nested loop.

<table>
<thead>
<tr>
<th>Sequence Class</th>
<th>Context</th>
<th>Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Contiguous</td>
<td>Inner</td>
<td>Privatization</td>
</tr>
<tr>
<td>2 Increasing</td>
<td>Outer</td>
<td>Independence</td>
</tr>
<tr>
<td>3 Contiguous</td>
<td>Outer</td>
<td>Efficient parallel code</td>
</tr>
</tbody>
</table>


loop if it is contiguous within every iteration and, for any iteration \( i \), its image over all iterations up to \( i \) is contiguous (Fig. 5(b)). It is consecutive in a loop if it is both contiguous and increasing in the loop (Fig. 5(c)). These definitions can be extended to strided memory access. These properties have to be proved true across all control paths.

We use VEG information to measure and compare the extent of memory reference sets and recurrence steps. This analysis is control-flow sensitive. In order to prove a sequence contiguous, we show that on all paths, and under the same or implied conditions the step of induction variable (obtained from the VEG) is smaller or equal to the span of the memory reference, at the loop level.

### 3.3. Classic Compiler Optimizations

Let us assume that we want to parallelize the outer loop of the nested loops Outer and Inner. Table 2 presents the overall use of memory reference sequence classification in privatization and data dependence analysis.

**Dataflow Analysis.** We can use the WF, RO, and RW sets to prove general dataflow relations. For instance, a WF followed by a RO represents a def-use edge with weight WF \( \cap \) RO. This information can be used in transformations such as constant propagation. In the example in Fig. 1, we can prove that there is a def-use edge between lines 8 and 13 on array A, with weight \([old+1:p_4]\). We can thus propagate all constant array values at offsets within this range.

**Privatization.** The privatization transformation benefits from memory reference sequence classification indirectly. The refined WF, RO, and RW sets for Inner will result in refined ROd, WFi, and RWd sets for Outer, leading to more opportunities for privatization. This corresponds to edge 6 in Fig. 6, and to row 1 in Table 2.

**Dependence Analysis.** Let us assume that we have the descriptors ROd, WFi, and RWd for Outer. If we can find a memory reference sequence \( d \) that includes them and is increasing in Outer, then there can exist no cross iteration data dependences. This corresponds to edges 3 and 5 in Fig. 6, and to row 2 in Table 2.

### 3.4. Recognition of Pushbacks and Other Parallelizable Prefix Computations

Many programs access arrays in loops according to patterns that are determined by loop induction variables. Even though induction variables are computed by recurrences, there are many important cases in which such loops can be executed in parallel. First, necessary conditions are that (i) there should be no data dependences between iterations of the loop except those involving the induction variable, (ii) there is no dependence cycle between the induction variable used as an address and the data computation, and (iii) it must be possible to compute the values taken on by the induction variable in parallel. Two cases in which the induction values can be computed in parallel are when the induction recurrence has a closed form solution or when it is associative; in the former case parallelization is trivial and in the latter case it can be done using a parallel prefix type computation [16]. Parallel prefix typically consists of three stages: (i) compute the local prefix sums of the associative induction variable, (ii) compute the prefix sums of the induction variable across processors, (iii) use the results of the cross-processor phase to compute the corresponding global values of the local indices, and then copy out the contents of the local arrays to their corresponding offset in the global array.

[19] addresses loops that contain the pattern \( p = p+1 \); \( A(p) = ... \) and where \( p \) does not appear anywhere else in the loop body, and parallelize them using a technique named “array-splitting,” which is essentially a prefix computation.

In this work, we use the VEG to extend the applicability of the parallel prefix parallelization to more general types of loops that cannot be analyzed using pattern matching techniques alone. In particular, for loops with induction variables with no closed form solution, we impose the condition that the induction variable can only be used as an address into an array, i.e., it does not contribute to the global data and/or control flow of the loop. In other words, if the induction variable is assigned to a shared variable or controls the execution of the program (e.g., used as an absolute inner loop bound) we will take the conservative approach and not parallelize it. An exception is made for the case when the value of the recurrence is used to test loop termination.

**Pushback sequences.** We first consider loops which compute so-called pushback sequences that are generally defined as a sequence of consecutive write-first (WF) reference sets. In the following, we describe how we have used information provided by the VEG to extend the applicability of parallel prefix parallelization to pushback sequences. For illustration we use the code example in Fig. 4 which effectively performs a pushback on array A.

a) References to the pushback array have to be WF only. This implies that read accesses, to the array covered by the WF are allowed in any order. The WF set is computed accu-
rately by the VEG improved MCA and thus qualifies more loops for parallelization. In the example in Fig. 4 we can see that the read at line 2 in function `ten` is always covered by a write in a previous iteration of the loop at line 2 in subroutine `build`, but within the same iteration of the loop in program `main`.

b) Most previous techniques analyze the patterns in which the induction variable appears, and from that try to infer which array addresses are used; this only works if there is a very simple (e.g., identity) relation between between induction values and array indices. We can qualify more loops as pushbacks since we can use VEG enhanced MCA to analyze more complex functions of the induction variable and determine if the resulting index sequence satisfies the $step \leq length^2$ condition.

c) Information provided by the VEG can help us identify cases in which the induction variable does not contribute to the control flow, even if it would appear that it does using pattern matching techniques. For instance, in Fig. 4 the reference to $A$ at line 2 in function `ten` is through $j$, which is the index of the loop at line $I$. This loop has recurrence values as bounds. The looping statement can be normalized as $DO \ k = 1, e-b$. We use the VEG and evaluate the (e-b) loop bound and find that it does not depend on the induction variable of the outer loop, i.e., that we do not use the value of the induction variables of the loop we are considering (we use a local value).

d) We can parallelize loops where the recurrence value is also used as an early termination condition. Such cases are common for error checks such as stack overflow which usually result in premature loop exits. We execute the parallel prefix speculatively [23], compute the final value of the recurrence variable (before the termination) and then use it to copy out only the section of the private arrays that fits in the correct bounds.

**Other Sequences.** Using VEG enhanced MCA we can parallelize additional sequences with parallel prefix. Here are some interesting sequences we can recognize:

a) A sequence whose index is generated by a simple associative recurrence with any positive step such that $step > length$. In this case, the copy out phase will require that the computation of the indices into the global array be done in a more complex manner than for a pushback. Instead of using ranges of global addresses we have to compute them individually.

b) Sequences whose index is generated by a more complex associative induction of some form $v = f(v, k)$ where $f$ is an associative operator. In this case, VEG enhanced MCA can be used to guide the application of the set operations. (Although it is true the set operations themselves will be more complex, that is a symbolic manipulation problem that is beyond the scope of this paper.)

It is interesting to remark that when we do not deal with a simple pushback sequence, the parallel prefix computation of the recurrence value and the actual computation of the loop must be done, conceptually, in separate stages. Sometimes it is beneficial to perform in the local stage only the computation of the recurrence values and leave the remainder of the loop computation for the third phase of the parallel prefix. Other times, when the distribution of the recurrence computation implies a large amount of work duplication, it is beneficial to compute everything in the first phase in private storage and leave the actual address computation and copy out for the third phase. The compiler can use a simple work evaluation model to decide between the two alternatives.

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2 The $step$ of the recurrence versus the $length$ of the memory access.
### Table 3. Loops parallelized. CP = Conditional Pushback, SL(U) = Stack Lookup (and Update), P-CW = Privatization based on Contiguous Writes, P-VEG = Privatization using the VEG directly.

<table>
<thead>
<tr>
<th>Program</th>
<th>Loop</th>
<th>Seq. %</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRACK</td>
<td>EXTEND,do400</td>
<td>15-65</td>
<td>CP-SLU, P-CW</td>
</tr>
<tr>
<td></td>
<td>FPTRAK,do300</td>
<td>4-50</td>
<td>CP-SL</td>
</tr>
<tr>
<td></td>
<td>GETDAT,do300</td>
<td>1-5</td>
<td>CP-SLU, P-CW</td>
</tr>
<tr>
<td>P3M</td>
<td>PP,do100</td>
<td>52</td>
<td>P-CW, P-VEG</td>
</tr>
<tr>
<td></td>
<td>SUBPP,do140</td>
<td>9</td>
<td>P-CW</td>
</tr>
<tr>
<td>BDNA</td>
<td>ACTFOR,do240</td>
<td>29</td>
<td>P-VEG</td>
</tr>
<tr>
<td>MDLJDP2</td>
<td>JLOOPB,do20</td>
<td>12</td>
<td>CP</td>
</tr>
<tr>
<td>ADM</td>
<td>DKZMH,do60</td>
<td>6</td>
<td>P-CW</td>
</tr>
<tr>
<td>QCD</td>
<td>QQQLPS,do21</td>
<td>&lt; 1</td>
<td>CP</td>
</tr>
<tr>
<td>DYFESM</td>
<td>SETCOL,do1</td>
<td>&lt; 1</td>
<td>CP</td>
</tr>
<tr>
<td>HYDRO2D</td>
<td>WNFL,do10</td>
<td>&lt; 1</td>
<td>CP</td>
</tr>
</tbody>
</table>

#### 4. Experimental Results

Hybrid Analysis [25] integrates compile-time and runtime analysis of memory reference patterns. Its static part consists mainly of a framework for aggregation of memory references using the compact RT_L,MAD memory location set representation. This framework is used to perform Memory Classification Analysis which is used for automatic parallelization. We have integrated the information produced by VEGs in this framework – Fig. 6(a). Fig. 6(b) shows the relation between three levels of abstraction in the analysis process. High-level routines such as dependence analysis relies on memory reference set operations (such as intersection) and on the recognition of increasing memory reference sequences. These operations rely heavily on VEG information, such as step, range, or logical inferences. We implemented the VEG and integrated it with the Hybrid Analysis pass in Polaris.

Table 3 presents our results over codes TRACK, BDNA, QCD, ADM and DYFESM from the PERFECT benchmark suite, P3M from the NCSA suite, and HYDRO2D and MDLJDP2 are from SPEC92. The third column shows the percentage of the total sequential execution of the program spent in the loop. The parallelization of these loops is crucial to the overall performance improvement in TRACK, BDNA, ADM, P3M, and MDLJDP2. Although our new techniques can parallelize a larger number of loops, we only display results in addition to the ones obtained using traditional analysis techniques.

Seven out of the eleven parallelized loops were conditional pushbacks. The cases in TRACK are the most difficult as the arrays are not used as a stack at statement level, but only at the whole loop body level. We are not aware of any other static analysis that can parallelize any of these three loops. Six out of eleven loops required privatization analysis based on either contiguous writes or VEG information directly (value ranges).

Loop BDNA/ACTFOR,do240 contains an inner loop that fills an index array ind with values within range [1:1], where i is the index of the outer loop. These values are then used to index a read operation on an array xdt. Since array xdt is first written in every iteration of the outer loop from i to i, this write covers all successive reads from xdt(ind(i)). The read pattern in ind(i) is found to be completely contained in [1:1] based on the VEG range approximation for ind, which proves xdt privatizable. This pattern also appears on some arrays in P3M/PP,do100.

We also ran the analysis on the Barnes-Hut code TREE from the University of Hawaii in order to compare our results to previous work reported in [18]. This is an interesting case of an array that is used as a stack (push and pop operations) within an iteration of a loop, and is thus privatizable. However, the loop cannot contain cross-iteration dependencies because the stack array is a local variable in a subroutine treewalk which is called from within the loop. Even if the code were inlined, our VEG-enhanced MCA would find the array privatizable in the outer loop.

**ADM/DKZMH,do60:**

The loops at line 1 and 2 in Fig. 7 can be parallelized if we can show that array A is privatizable. We show that A has no exposed reads for the context between lines 3-14.

At lines 3-11, \( W = [1:p2] \cup [p3:nz] \). The distance between \( p2 \) and \( p3 \) is the value range for variable \( inc_3 \). This range was found by the VEG for subroutine \( sr \) to be \([0:1]\). This implies \( p2+p1 \geq p3 \), so \( W = [1:nz] \). At lines 3-14, \( RO = RO - W = [1:nz] - [1:nz] = 0 \). \( W = [1:nz] \).

Figure 7. Code extracted from DKZMH,do60.
TRACK/EXTEND_do400:
This loop is our most complex case, and was presented in detail as the loop in program main in Fig. 4. It consists of pushbacks performed in an inner loop. Another loop, inner to both of them, reads backwards the elements that were pushed within the same iteration of the outermost loop and, based on some condition, may modify some of these locations. It is crucial to prove that the access within an iteration of the outer loop is confined to locations that were pushed within the same iteration. In addition to the problems discussed with relation to Fig. 4, this loop presents another problem that can only be solved using a conditionally pruned VEG.

This innermost loop is shown in Fig. 8. The range of the elements that have been pushed back within the current iteration of the outermost loop is \([old:p]\). The write reference at line 10 is at offset same_2. We must prove that same_2 is within \([old:p]\). A simple query on the range of values for same_0 on the VEG returns a range of \([1:p]\) because it has to take into account the possibility that same_2 was not defined in the loop at line 2. The value could have come on edge same_0 \(\rightarrow\) same_2. Since same_0 could have carried a value from a previous iteration of the outermost loop, same_2 might not be confined to what was pushed in the current iteration. However, this edge is removed during the pruning of the VEG based on condition flag_3.EQ.1. Since a value for flag_3 could have come only on edge flag_2 \(\rightarrow\) flag_3, and since the pair (flag_2, flag_3) corresponds to the same control flow edge as (same_1, same_2) (based on GSA \(\gamma\) predicates), we can remove edge same_0 \(\rightarrow\) same_2. The VEG now evaluates the range of same_2 to \([old:p]\) for the use at the statement at line 10.

5. Related Work

Recurrence Recognition, Classification, and Parallelization. [1, 26, 29, 8] present the automatic recognition and classification of general recurrences. The idea of a value graph was introduced in [24]. Although similar to the SSA graph [29], the VEG adds more power and functionality to the representation: closure for the meet-over-paths operator using ranges and accuracy by pruning based on conditionals. [6] discusses the parallelization of linear recurrences. [5] and [7] present the recognition and parallelization of certain classes of recurrences but do not address cases when memory is referenced using the values of the recurrence. We express recurrence functions as paths in VEGs. Although in theory these techniques could cover more cases, in practice they are limited to recurrence formulae consisting of linear algebraic expressions coupled with conditionals. Since some of the edges in the VEG represent algebraic relations and others represent conditional execution, we can also represent this mix of linear functions and conditionals.

Analysis of Memory Referenced by Recurrences without Closed Forms. [13] and [20] found more closed forms for classes of recurrences that had not been commonly recognized/substituted by compilers. [9] presents the parallelization of loop nests that may contain recurrences by flattening the nests into single loops and pre-computing the recurrences in inspector loops. This method may not be feasible when the recurrence depends on computation within the loop itself. [12] presents the use of monotonicity in reducing the number of bound checks for arrays referenced using a recurrence without closed form.

Let us follow (by row in Table 4) a comparison between our framework and the most recent work on the parallelization of loops that reference memory through recurrences without closed forms [11, 18, 30, 31].
solutions by associating them with graph paths. These paths contain explicit evolution and control information (by using GSA). The VEG can model recurrences defined using multiple variables, unlike previous representations that rely on the statement-level pattern \( i = i + \text{exp} \). The VEGs are pruned based on ranges extracted from conditional values, which leads to closer value ranges. The static parallelization of loop \( \text{EXTEND}_{\text{do}400} \) can only be decided on this pruned graph, and has not been reported before.

7, 8. [18, 30, 31] require that arrays be unidimensional and that the index expression consist of exactly the recurrence variable. The recurrence variable cannot appear in the loop text except for the recurrence statements and as an array index. Our framework is more flexible: we analyze partially aggregated generic memory descriptors that represent the reference pattern in a single statement, an inner loops or a whole subprogram uniformly. Loop \( \text{DKZMH}_{\text{do}60} \), and loops \( \text{EXTEND}_{\text{do}400} \) and \( \text{FPTRAK}_{\text{do}300} \) reference memory in inner loops and via subroutines; some arrays are two-dimensional; in one case array elements are seen as scalars inside a called subprogram; in a few cases, the recurrence variable appears in the bounds of an inner loop, while the actual array index expression is the loop index.

9. We present the parallelization of pushback sequences in a more general case than the one presented in [18] where the important loops \( \text{EXTEND}_{\text{do}400} \), \( \text{FPTRAK}_{\text{do}300} \) and \( \text{GETDAT}_{\text{do}300} \) from TRACK could not be parallelized.

6. Limitations and Future Work

At this point we only support one evolution type within a single graph. This allows us to compose evolutions along paths by performing simple range arithmetic. We are planning to investigate the need for an evolution graph that contains multiple types of evolutions and the algorithmic complications involved.

For now, we treat arrays as scalars. We are planning to investigate the use of array dataflow information produced by MCA to create more expressive value evolution graphs.

We are also looking into further applications of value evolution graphs to the GSA path technique. Preliminary results show that, with minor improvement, we could solve more complex problems such as the parallelization of loop \( \text{INTERF}_{\text{do}1000} \) in code MDG from the PERFECT suite.

References


