Toggle PRM: A Coordinated Mapping of C-free and C-obstacle in Arbitrary Dimension

Jory Denny and Nancy M. Amato

Abstract Motion planning has received much attention over the past 40 years. More than 15 years have passed since the introduction of the successful sampling-based approach known as the Probabilistic RoadMap Method (PRM). PRM and its many variants have demonstrated great success for some high-dimensional problems, but they all have some level of difficulty in the presence of narrow passages. Recently, an approach called Toggle PRM has been introduced whose performance does not degrade for 2-dimensional problems with narrow passages. In Toggle PRM, a simultaneous, coordinated mapping of both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$ is performed and every connection attempt augments one of the maps – either validating an edge in the current space or adding a configuration 'witnessing' the connection failure to the other space. In this paper, we generalize Toggle PRM to d-dimensions and show that the benefits of mapping both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$ continue to hold in higher dimensions. In particular, we introduce a new narrow passage characterization, α-ε-separable narrow passages, which describes the types of passages that can be successfully mapped by Toggle PRM. Intuitively, α-ε-separable narrow passages are arbitrarily narrow regions of $\mathbb{C}_{free}$ that separate regions of $\mathbb{C}_{obst}$, at least locally, such as hallways in an office building. We experimentally compare Toggle PRM with other methods in a variety of scenarios with different types of narrow passages and robots with up to 16 DOF.

1 Introduction

In robotics, planning a valid (e.g., collision-free) path for a movable object (robot) is challenging. *Motion planning*, as this is commonly called, has been thoroughly studied [23] and has been shown to be PSPACE-hard. Motion planning has broad

Parasol Lab, Department of Computer Science and Engineering, Texas A&M University, College Station, TX, USA, e-mail: {jdenny, amato}@cse.tamu.edu

use outside robotics in areas such as gaming/virtual reality [14] and bioinformatics [25]. Hence, it is vital to find computationally efficient solutions to this problem.

Sampling-based planners [12] were a major breakthrough in motion planning. These algorithms were able to solve many previously unsolved problems, especially for high-dimensional configuration space ($\mathbb{C}_{space}$). Sampling-based motion planners explore $\mathbb{C}_{space}$ and store configurations and some information about their connectivity in $\mathbb{C}_{free}$. One class of planners, *Probabilistic RoadMap* methods (PRMs) [12], build a roadmap (graph) representing the connectivity of $\mathbb{C}_{free}$. The first phase in this process, *node generation*, is where collision-free configurations are sampled and added as nodes to the roadmap. In the second phase, *node connection*, neighboring nodes are selected by a *distance metric* as potential candidates for connection. Then, simple *local planners* attempt connections between the selected nodes; successful connections are represented as roadmap edges. While these methods have been shown to be *probabilistically complete*, narrow passages, or small volume regions of free space ($\mathbb{C}_{free}$), remain difficult for them to map. In particular, it has been shown that the volume of such passages impacts the efficiency of a sampling-based planner [11]. In simple terms, the probability of generating a node within the narrow passage is directly related to the volume of the passage itself. Thus, the tighter the corridor is, the more difficult it is to generate samples in it. Section 2 briefly discusses the many attempts to address this weakness of sampling-based planning methods [1, 3, 26, 10].

Recently, a new sampling-based motion planning strategy that maps both free space ($\mathbb{C}_{free}$) and obstacle space ($\mathbb{C}_{obst}$) in a simultaneous, coordinated fashion has shown to be useful in 2-dimensional problem spaces for both roadmap construction (Toggle PRM [6]) and local planning (Toggle LP [7]). In particular, in Toggle PRM, coordinated learning of both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$ was shown to theoretically guarantee a higher probability of sampling within narrow regions of $\mathbb{C}_{space}$ for 2 DOF problems, while Toggle LP extended local planning attempts into a 2D triangular subspace of $\mathbb{C}_{space}$ to yield better connectivity of roadmaps as opposed to the traditional straight-line connection [12]. The intuition behind Toggle PRM and the main departure from traditional PRMs is the philosophy that every connection attempt, successful or not, reveals information about connectivity in one of the spaces and hence augments one of the maps – either validating an edge in the current space or discovering a configuration in the other space 'witnessing' the connection failure. For example, a failed connection between two collision configurations on either side of a narrow passage would lead to the discovery of a configuration in the narrow passage. Connection attempts between connected components of a roadmap also become increasingly interesting for Toggle PRM. Ultimately, Toggle PRM might be able to induce disconnectivity for planning problems, something that is not feasible for PRMs (see Figure 1). As shown in [6], utilizing the witnesses of connection failures can greatly accelerate the discovery and mapping of narrow passages for 2 DOF problems.

This paper further develops the Toggle PRM philosophy for sampling-based motion planning and generalizes it to higher dimensions. We show that the theoretical and experimental benefits of a coordinated mapping of both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$, demonstrated in [6], continue to hold in higher dimensions. In particular, we show that

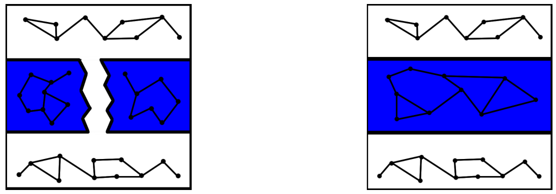


Fig. 1: A map of $\mathbb{C}_{obst}$ can reveal important connectivity information possibly differentiating between a problem with (left) and without (right) a narrow passage.

Toggle PRM results in significant improvements to the effectiveness and efficiency of sampling-based planners, for a particular class of narrow passages, α-ε-separable narrow passages, that we define here. Particular contributions of this work include:

- Generalizing Toggle PRM to arbitrary dimension.
- Defining a new characterization of narrow passages, α-ε-separable narrow passages, that can be mapped well by Toggle PRM.
- Providing experimental results for problems that confirm Toggle PRM's ease of mapping α-ε-separable narrow passages and demonstrate improvement over other PRM methods.

2 Preliminaries and Related Work

Motion Planning Preliminaries. A robot is a movable object whose position and orientation can be described by n parameters, or *degrees of freedom* (DOFs), each corresponding to an object component (e.g., object positions, object orientations, link angles, link displacements). Hence, a robot's placement, or configuration, can be uniquely described by a point $(x_1, x_2, ..., x_n)$ in an n dimensional space (x_i being the ith DOF). This space, consisting of all possible robot configurations (feasible or not) is called *configuration space* ($\mathbb{C}_{space}$) [16]. The subset of all feasible configurations is the *free space* ($\mathbb{C}_{free}$), while the union of the unfeasible configurations is the *blocked* or *obstacle space* ($\mathbb{C}_{obst}$). Thus, the motion planning problem becomes that of finding a continuous trajectory in $\mathbb{C}_{free}$ connecting the points in $\mathbb{C}_{free}$ corresponding to the start and the goal configurations. In general, it is intractable to compute explicit $\mathbb{C}_{obst}$ boundaries [23], but we can often determine whether a configuration is feasible or not quite efficiently, e.g., by performing a collision detection (CD) test in the *workspace*, the robot's natural space.

PRM variants. PRMs have known difficulty in solving narrow passage problems [11]. To address this deficit, many variants have been proposed, e.g., [21, 2, 13]. The performance of these methods is dependent on the problem instance. Below is a description of the PRM variants most related to this work.

Several methods have been proposed that attempt to generate samples on or near $\mathbb{C}_{obst}$ surfaces. The first, Obstacle-Based PRM (OBPRM) [1], is efficient at solving problems in practice but does not sample with a known density on surfaces and thus its theoretical benefits are not understood. Recently, a new method called uniform OBPRM (UOBPRM) [9] has been proposed that guarantees a uniform distribution of samples on $\mathbb{C}_{obst}$ surfaces, albeit for a larger computational cost than OBPRM. Gaussian PRM [3] attempts to find configurations near obstacles by generating two samples at a Gaussian d away from each other and tests if it straddles the $\mathbb{C}_{obst}$ boundary. Bridge Test PRM [10] is similar to Gaussian PRM. It is a filtering technique that attempts to generate two invalid configurations, and only if this is successful it tests the midpoint for validity. Hence, this test attempts to bridge the gap and generate configurations in narrow passages. However, this test may fail many times before successfully bridging a gap, depending on the narrow passage.

Medial Axis PRM (MAPRM) [26, 15] pushes randomly sampled configurations to the medial axis. It provably improves the probability of sampling in most narrow passages over uniform random sampling, but is computationally intensive even when approximations are used.

Toggle PRM [6] performs a coordinated mapping of $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$. It was shown theoretically and experimentally to perform better than uniform random sampling, and most of the methods described here, in 2D $\mathbb{C}_{space}$. Toggle Local Planning [7] used these ideas to extend local planning into a 2-dimensional triangular subspace of $\mathbb{C}_{space}$, which was seen to result in improved connectivity over the traditional straight-line local planner [12]. Toggle PRM differs from Bridge Test by not only testing an entire line segment for crossing narrow passages but also by fully mapping $\mathbb{C}_{obst}$, including connections.

Variants utilizing collision information. There have been many improvements upon PRM that use information gain or learning techniques to guide sampling towards specific regions of the environment. Many of these methods utilize information about nodes in collision, however none of them create a map of $\mathbb{C}_{obst}$. This section briefly describes some of these methods.

There is a class of motion planning techniques called Feature Sensitive Motion Planning [18, 20, 24, 27] that apply a divide-and-conquer approach by breaking up the environment into regions, solving each region, and combining the solutions. Although these techniques do not map $\mathbb{C}_{obst}$ these approaches often use information from samples, both valid and invalid to find appropriate regions.

Other efforts, such as [4, 5, 19, 22], seek to model $\mathbb{C}_{space}$ to more efficiently map $\mathbb{C}_{free}$. In [4, 5] a machine learning technique, locally weighted regression, is used to create an approximate model of $\mathbb{C}_{space}$ that biases sampling toward areas which improve the model, called active sampling.

3 Toggle PRM

In this section, we provide an overview of the Toggle PRM algorithm [6] and its theoretical benefits. To generalize the algorithm to arbitrary dimension, certain aspects of higher dimensionality must be considered, i.e., connection attempts along a one-

Algorithm 1 *Toggle PRM*. Local planners and connectors are modified to return configurations from failed connection attempts.

```
Input: Sampler s
Roadmaps G_free, G_obst = ∅
while !done do
    Queue q ← s.Sample()
    while ¬q.isEmpty() do
        Configuration n = q.dequeue()
        if n is valid then
            G_free.AddNode(n)
            collisionNodes ← TogglePRMConnect(G_free)
            q.enqueue(collisionNodes)
        else {n is invalid, and local planners "validate" paths through C_obst}
            Toggle Validity
            G_obst.AddNode(n)
            validNodes ← TogglePRMConnect(G_obst)
            q.enqueue(validNodes)
            Toggle Validity
        end if
    end while
end while
```

dimensional line through $\mathbb{C}_{space}$ (e.g., straight-line local planning) are less likely to intersect a small volume of $\mathbb{C}_{space}$. To extend the algorithm to higher dimensions, issues such as this must be addressed. In the following, we first sketch the Toggle PRM algorithm, including a new connection strategy that is better at discovering and mapping certain types of "locally separable" small volume regions of $\mathbb{C}_{space}$. We then formalize the concept of locally separable by defining α-ε-separable narrow passages and proving that they can be mapped well by Toggle PRM.

Algorithm. An overview of *Toggle PRM* is shown in Algorithm 1. The major difference from traditional PRM methods is that roadmaps of both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$ are constructed. The strategy is based on the fact that each successful connection attempt reveals connectivity information about one of the spaces, while each unsuccessful attempt witnesses a crossing of the opposite space. In particular, the local planning method used to verify edge connections returns a "witness" configuration if the connection attempt fails, which is used to augment the opposite spaces map.

The algorithm uses a sampler to generate nodes (e.g., uniform random sampling), and a connector, which is a combination of a local planner (e.g., straight-line) and a distance metric (e.g., Euclidean), to find the neighboring nodes to which to attempt connections. Algorithm 1 differs from the original description of Toggle PRM in the connection phase, described in Algorithm 2, which uses a heuristic particularly designed for mapping α-ε-separable narrow passages. We start by initializing two roadmaps, G_{free} and G_{obst}. While the map construction is not done (e.g., until the map is a certain size, a query is solved, etc.), then the following happens: samples are generated in both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$, and connections are attempted in both maps using Algorithm 2. Witnesses returned by failed connections are added to the opposite space's roadmap. When connections are attempted in $\mathbb{C}_{obst}$, toggling the

Algorithm 2 *TogglePRMConnect*. Local planners and connectors are modified to return configurations from failed connection attempts.

Input: Roadmap G
Output: Collision Nodes C

for all Newly added nodes n **do**
  repeat
    Attempt connection from n to a neighbor b in a different CC of G as n
    if Connection attempt fails **then**
      Add witness of failed connection attempt to C
    else
      $G.AddEdge(n,b)$
    end if
  until Connection attempt fails
end for
return C

meaning of validity is necessary for the local planner to "validate" paths through $\mathbb{C}_{obst}$, i.e., all configurations along a local plan must be located in $\mathbb{C}_{obst}$. The algorithm repeats until the stopping criterion is reached, or it is shown to be unreachable. The algorithm uses a *queue* to incrementally build the roadmap.

The major difference from the 2D version of Toggle PRM is the connection phase. While any connection algorithm could be used, we introduce a new strategy designed to do well when connections would cross narrow passages that separate $\mathbb{C}_{obst}$. Selecting appropriate neighbors for connection is crucial since it will either allow you to learn about the connectivity of the current space, i.e., successful connection attempts, or about the opposite space, i.e., failed connection attempts. However, if learning is biased towards the opposite space too much, we "flood" the queue, which effectively oversamples $\mathbb{C}_{space}$ by attempting too many wasteful (and costly) connections to similar nodes within the roadmaps. Ideally, we want to connect to the current space's roadmap *and* learn enough about the opposite space.

Toggle PRM uses a novel connection strategy shown in Algorithm 2 that attempts to connect unconnected nodes of a roadmap to new connected components. After a node merges into a connected component, connections are attempted to other connected components. This repeats until a failure is reached, i.e., Toggle PRM learns about the opposite space. In higher dimensions, we have found that connecting into the roadmap (in order of closest to furthest roadmap nodes) until a failed connection is found reveals enough connectivity information in a space without oversampling. Eventually, this connectivity information could lead to discovering disconnections in the planning space.

3.1 α-ε-separable Narrow Passages

In this section, we define a type of narrow passage that can be mapped well by Toggle PRM. Let $\mu(P)$ be a function which computes the hypervolume of a polytope P.

Let $\mathbb{C}_X$ be either $\mathbb{C}_{free}$ or $\mathbb{C}_{obst}$ and $\mathbb{C}_Y$ be the complement of $\mathbb{C}_X$. Definitions 1, 2, 3, and 4 describe necessary geometric objects for Theorem 1 and Corollaries 1, 2, and 3 to prove that Toggle PRM performs better than uniform random sampling in these objects. Finally, Definition 5 classifies an α-ε-separable narrow passage, and Proposition 1 proves improvement over uniform random sampling.

Definition 1. A *separable polytope* (Figure 2(a)) is a polytope P that is composed of three distinct polytopes; two outer polytopes $A_1, A_2 \subset \mathbb{C}_X$ and one inner polytope $B \subseteq \mathbb{C}_Y$ such that any line segment between points $a_1 \in A_1$ and $a_2 \in A_2$ lies completely in P and crosses B.

Definition 2. Let $\varepsilon \in [0,1)$ be arbitrarily close to 0. A separable polytope $S = \{A_1, B, A_2\}$ is *ε-separable* (Figure 2(b)) if $\mu(B) \leq \varepsilon\mu(S)$.

Definition 3. Let $\alpha \in (0,1]$. An α-separable polytope P (Figure 2(c)) is a polytope composed of two distinct polytopes $A \subseteq \mathbb{C}_X$ and $B \subseteq \mathbb{C}_Y$ such that there exists a separable polytope $S \subseteq P$, such that $\mu(S) \geq \alpha\mu(P)$.

Definition 4. An α-separable polytope P is *α-ε-separable* (Figure 2(d)) if the separating polytope $S \subseteq P$ is ε-separable.

These definitions describe various shapes which could be components of $\mathbb{C}_{space}$. The inner sections of the polytopes typically correspond to narrow passages and outer sections are components of $\mathbb{C}_{obst}$. Together, they lay the foundation for defining the types of narrow passages that can be mapped well with Toggle PRM.

Theorem 1. *Given two sampling attempts, Toggle PRM will sample in the inner section B of a separable polytope $S = \{A_1, B, A_2\}$ with probability*

$$P = \left(1 - \left(1 - \frac{\mu(B)}{\mu(\mathbb{C}_{space})}\right)^2\right) + \binom{2}{1}\frac{\mu(A_1)\mu(A_2)}{\mu(\mathbb{C}_{space})^2}$$

Proof. For Toggle PRM to sample in B, either one of the two samples must lie in B (i.e., uniform sampling yields a sample), the first sample lies in A_1 and the second in A_2, or the first lies in A_2 and the second in A_1 (i.e., the connection attempt yields a sample). The probability to uniformly sample in B given two sampling attempts is

$$P_{uniform} = \left(1 - \left(1 - \frac{\mu(B)}{\mu(\mathbb{C}_{space})}\right)^2\right)$$

The probability to get one sample in A_1 and one in A_2 is

$$P_{local planner} = \binom{2}{1}\frac{\mu(A_1)\mu(A_2)}{\mu(\mathbb{C}_{space})^2}$$

□

Corollary 1. *Given two sampling attempts, Toggle PRM will sample in section B of an ε-separable polytope $S = \{A_1, B, A_2\}$, with $\varepsilon = 0$, with probability*

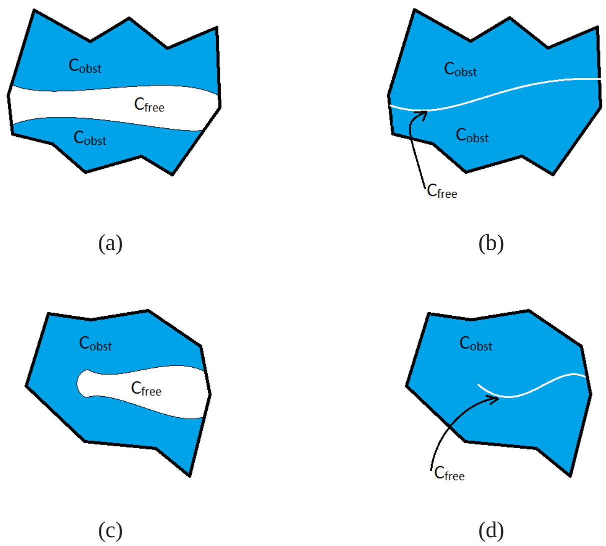


Fig. 2: Examples of various separable polytopes. (a) Separable polytope $S = \{A_1, A_2 \subset \mathbb{C}_{obst}, B \subseteq \mathbb{C}_{free}\}$. (b) ε-separable polytope $S = \{A_1, A_2 \subset \mathbb{C}_{obst}, B \subseteq \mathbb{C}_{free}\}$. (c) α-separable polytope $P = \{A \subseteq \mathbb{C}_{obst}, B \subseteq \mathbb{C}_{free}\}$. (d) α-ε-separable polytope $P = \{A \subseteq \mathbb{C}_{obst}, B \subseteq \mathbb{C}_{free}\}$.

$$P = \binom{2}{1} \frac{\mu(A_1)\mu(A_2)}{\mu(\mathbb{C}_{space})^2}$$

Proof. If $\varepsilon = 0$ then $\mu(B) = 0$ and the probability to uniformly sample within B is 0. Thus, the probability from Theorem 1 collapses to

$$P = \binom{2}{1} \frac{\mu(A_1)\mu(A_2)}{\mu(\mathbb{C}_{space})^2}$$

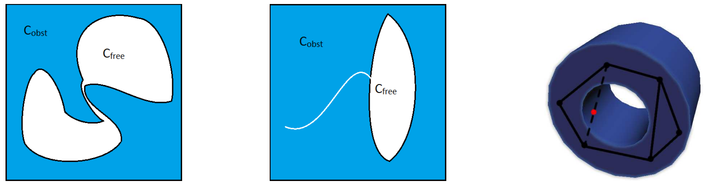
□

Corollary 2. *Given two sampling attempts, Toggle PRM will sample in the inner section B of an α-separable polytope $Q = \{A_1, B, A_2\}$ with non-zero probability.*

Proof. Let $S = \{A_1, B, A_2\} \subseteq Q$ be the largest separable polytope within Q. Since the hypervolume of S is greater than an α ratio of the hypervolume of Q, Theorem 1 implies non-zero probability of sampling in B since α is strictly greater than 0. □

Corollary 3. *Given two sampling attempts, Toggle PRM will sample in the inner section B of an α-ε-separable polytope $Q = \{A_1, B, A_2\}$ with non-zero probability.*

Definition 5. A narrow passage N is *α-ε-separable* if there exists an α-ε-separable polytope $Q = \{A_1, B, A_2\}$, where $A \subseteq \mathbb{C}_{obst}$ and $B \equiv N$.



(a) Passage 1 (b) Passage 2 (c) Passage 3

Fig. 3: Examples of α-ε-separable narrow passages (a and b) and non-separable narrow passages (c).

Proposition 1. *Given the same number of sampling attempts, Toggle PRM will sample in an* α-ε*-separable narrow passage with probability greater than that of uniform sampling.*

Proof. By definition, the narrow passage N is a component of a larger α-ε-separable polytope. By Theorem 1 and Corollaries 1, 2, and 3, Toggle PRM will sample in N with non-zero probability greater than that of uniform sampling. □

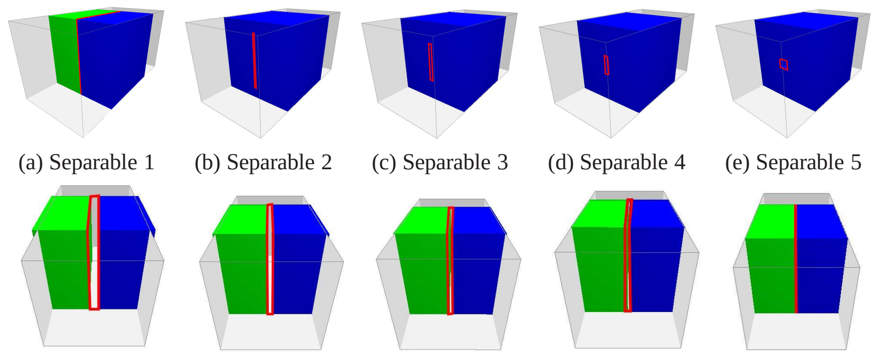
It can be seen that the connection heuristic described in Algorithm 2 performs well in these narrow passages. For example, in a separable polytope $S = \{A_1, B, A_2\}$, if connected components develop in both A_1 and A_2, then connections between them will generate samples in B. Additionally, there is a compounding affect in Toggle PRM that promotes sampling within narrow passages in both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$. For example, if the middle section of a separable polytope is in $\mathbb{C}_{obst}$, then a Toggle PRM may generate a sample in $\mathbb{C}_{obst}$ and the connections from this sample to other samples in $\mathbb{C}_{obst}$ will map the passage.

Intuitively, α-ε-separable narrow passages almost separate the surrounding space. Essentially, if the narrow passage is a hyperplane of dimension $d-1$, where d is the dimension of $\mathbb{C}_{space}$, then Toggle PRM should be effective in mapping the narrow passage. Examples of α-ε-separable narrow passages are shown in Figure 3(a) and (b). However, Toggle PRM might have difficulty mapping problems such as a cylindrical $\mathbb{C}_{space}$ (3D shown in Figure 3(c)) with a hole through it (a 1D line), as it is not separable.

4 Experiments

In this section, we describe experiments conducted to test the effectiveness of Toggle PRM. First we describe the experimental setup used in the various experiments. Then we compare Toggle PRM to other common sampling-based methods in a variety of experiments ranging from varying narrow passage widths, surrounding obstacle volume widths, articulated linkages, and specific problems.

Experimental setup. Toggle PRM, uniform random sampling, medial-axis sampling (MAPRM), Gaussian sampling, bridge-test sampling, and obstacle-based sam-



(a) Separable 1 (b) Separable 2 (c) Separable 3 (d) Separable 4 (e) Separable 5

(f) Passage 1 (g) Passage 2 (h) Passage 3 (i) Passage 4 (j) Passage 5

Fig. 4: Simple environments that vary narrow passage shape while keeping the volume constant (Separable 1-5) or that vary the passage volume while keeping the shape fixed (Passage 1-5). Narrow passages are highlighted (red).

pling (OBPRM) were implemented using the C++ motion planning library developed by the Parasol Lab at Texas A&M University. RAPID [8] or VCLIP [17] were used for collision detection computations. RAPID is used for higher DOF experiments, while VCLIP is used on the control experiments presented on simple environments. In these planners, connections are attempted between each node and its k-nearest neighbors according to a distance metric; here we use $k = 5$, $\mathbb{C}_{space}$ Euclidean distance, and a simple straight-line local planner using a bisection evaluation strategy. The bisection evaluation strategy is commonly used because it usually identifies failures faster and hence reduces the amortized cost of failed connection attempts. We chose $k = 5$ to keep the computation cost of connections relatively low. During connections, only one failure connection is allowed per node, i.e., up to k connections will be attempted, but testing ceases after the first failure. This reduces overhead of connections for Toggle PRM, as well as allowing it to generalize to higher dimensions. Gaussian sampling is implemented as described in [3], with a Gaussian $d_{gauss} = min(NP_{width}, Obst_{width})$, where NP_{width} is the narrow passage width and $Obst_{width}$ is the narrowest width of all surrounding obstacles. d_{gauss} is chosen to find samples which straddle the edges of $\mathbb{C}_{obst}$ surrounding the narrow passage. Bridge-test sampling is implemented as described in [10], with $d_{bridge} = NP_{width} + d_{gauss}$ chosen so that samples can span the narrow passage. OBPRM is based on $\mathbb{C}_{space}$ validity changes and implemented as in [1]. MAPRM is implemented using exact clearance information in low DOF cases and approximate clearance information in higher DOF cases as described in [15].

A few important metrics are used as a basis of comparison of the methods. Firstly, the number of free nodes sampled in the narrow passage is used to compare a sampler's ability to generate nodes in varying narrow passages. In problem solving tasks, the number of nodes in the roadmap of $\mathbb{C}_{free}$, number of CD calls (as a metric of efficiency of the planner), and the percentage of queries solved is compared. All

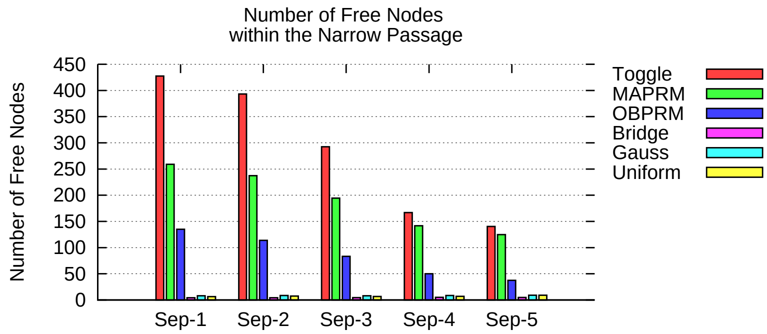


Fig. 5: The number of free nodes sampled in the narrow passages from 1000 attempts in the Separable environments for Toggle PRM (red), MAPRM (green), OBPRM (blue), bridge-test sampling (magenta), Gaussian sampling (cyan), and uniform sampling (yellow).

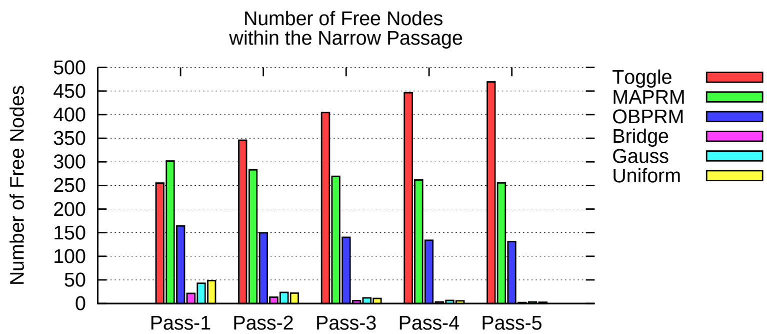


Fig. 6: The number of free nodes sampled in the narrow passages from 1000 attempts in the Passage environments for Toggle PRM (red), MAPRM (green), OBPRM (blue), bridge-test sampling (magenta), Gaussian sampling (cyan), and uniform sampling (yellow).

results are averaged over 10 runs for all experiments, if a planner is unable to solve all problem instances, averages over only problems solved is reported.

Controlled narrow passages. In this set of experiments, 1000 samples are attempted with each method (valid or invalid) in 10 simple environments that vary narrow passage volume or shape, using a point robot in 3-space.

The Separable environments (Separable 1-5 in Figure 4) have passages ranging from fully separable (Separable-1) to $\frac{1}{8}$ separable (Separable-5), by varying passage shape while keeping volume constant. The results in Figure 5 show that Toggle PRM outperforms the other methods in terms of successfully generating samples within the narrow passage. Additionally, in agreement with the analysis in Section 3.1, the effectiveness of Toggle PRM declines as the separability decreases. Indeed, this is true for all methods except for uniform, since its sampling density depends only on the volume.

The Passage environments (Passage 1-5 in Figure 4) vary the narrow passage volume from a width of 8 (Passage-1) to 0.5 (Passage-5), while keeping the shape fixed. The results in Figure 6 show that as the passage becomes narrower (ε decreases), Toggle PRM actually generates more samples in the narrow region while all other methods declined in performance. The improved performance of Toggle PRM can be explained as follows. Recall that, unlike other methods, Toggle PRM samples in both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$, and that the total number of nodes it generates is roughly constant over the various environments. Hence, these results indicate that as a region becomes more difficult to map in one space (the passage becomes narrower) then the surrounding region is easier to map, and moreover, the coordinated mapping of it will facilitate the discovery of samples in the difficult space.

Toggle PRM for articulated linkages. To show how Toggle PRM generalizes to higher dimensions, we compare Toggle PRM on a series of articulated linkage problems. The environment obstacles stay the same in each query problem, in which a query must traverse the small hole, as shown in Figure 7(a). The free base linkages vary from 3 links (8 DOF) to 11 links (16 DOF), as seen in Figure 7(b-f). Planners stop when either the query is solved or 1000 nodes have been sampled in $\mathbb{C}_{free}$. However, not all planners were able to solve example queries every time, so the percentage of queries solved is shown in Table 1. Results for both the number of free nodes and the number of CD calls required to solve the problem are shown in Figure 8.

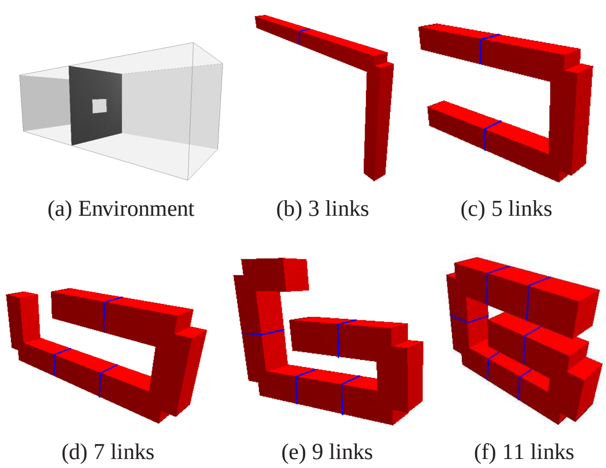
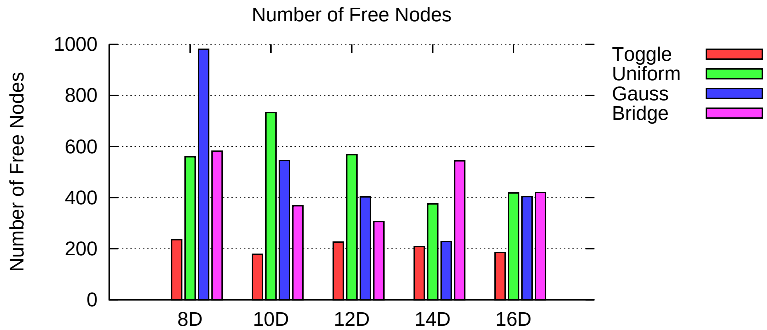
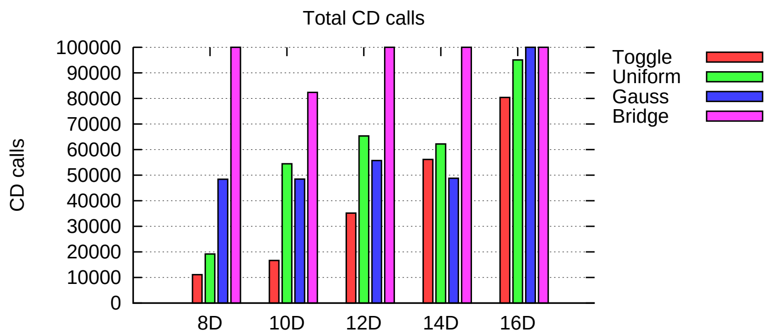


Fig. 7: Hook environment (a) along with 5 articulated linkage robots ranging from 3 links (b) to 11 links (f).

As shown in the results, Toggle PRM clearly solves more queries than the other planners. Toggle PRM typical can solve 100% of the queries, meaning its ability to solve problems efficiently, given a simple restriction such as was done in this



(a)



(b)

Fig. 8: Metrics required to solve queries for the various Hook problems for Toggle PRM(red), uniform sampling (green), Gaussian sampling (blue), bridge-test sampling (magenta)

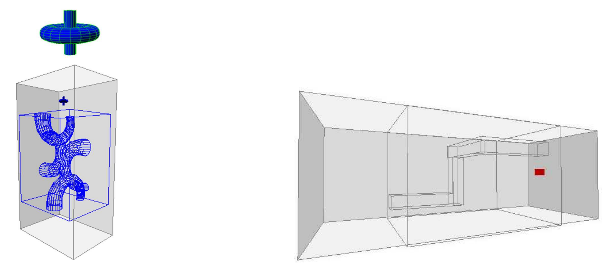
Linkage	Toggle	Uniform	Gauss	Bridge
8 DOF	**100**	30	10	40
10 DOF	**100**	50	60	40
12 DOF	**100**	40	60	70
14 DOF	**100**	60	40	80
16 DOF	**90**	40	40	30

Table 1: Percentage of successful planning attempts in Hook environments.

experiment (limit of 1000 nodes), is more robust. Excluding the 14 DOF case where Gaussian sampling seems more efficient than Toggle PRM, Toggle PRM typical reduces the computational cost for solving such problems, even in higher DOFs. In the 14 DOF case however, Toggle PRM still outperforms Gaussian sampling in terms of nodes needed and percentage of instances solved. Many times however, Toggle PRM has half of the CD calls as the other comparable methods.

As we can see, Toggle PRM has increased sampling density within various narrow passages. Additionally, we have shown how Toggle PRM performs as the DOF increases, such as with articulated linkages. Next, we present this improvement for more general problems.

Problem solving. In our last experiment, we emphasize the usefulness of Toggle PRM in a series of long tunnel environments, that a robot (3D rigid body - 6 DOF) must traverse. In these experiments, we limit a planner's roadmap nodes to 5000, success ratios are based upon this. The environments Maze-tunnel and Z-tunnel are shown in Figure 9. The results of these experiments are shown in Table 2.



(a) Maze-tunnel (b) Z-tunnel

Fig. 9: Solutions paths through these complicated environments must traverse through the narrow passage.

Environment	Maze-tunnel			Z-tunnel		
Planner	% Queries Solved	Free Nodes	CD Calls	% Queries Solved	Free Nodes	CD Calls
Toggle PRM	**100**	**420**	**1.37E5**	**100**	**318**	6.25E5
Uniform	60	4355	1.62E5	90	2500	6.27E5
MAPRM	100	720	4.99E6	100	361	9.07E5
Gaussian	100	2436	8.34E5	100	952	**4.47E5**
Bridge	60	2666	5.2E6	30	702	3.22E6
OBPRM	100	749	2.27E6	100	1441	8.1E5

Table 2: Results from tunnel problems. Toggle PRM outperforms the other planners.

As the results show, Toggle PRM is able to map $\mathbb{C}_{free}$ more efficiently than the other methods, typically using fewer CD calls and fewer overall nodes for both environments. In the Z-tunnel, even through Toggle PRM does not reduce the number of collision detection calls as much as Gaussian sampling, Toggle PRM reduces the number of nodes required to solve the problem. Toggle PRM could be made more efficient in this scenario if we extend the blocked region past the bounds of the environment. Moreover, Gaussian sampling requires a definition of the *d* value, making tuning an issue. Regardless, these other methods have shown merit, and we believe combining the use of these samplers might bring out the best of all methods, as in using Gaussian sampling to feed samples to Toggle PRM.

5 Conclusion

In this paper, we presented a generalized Toggle PRM paradigm which extends naturally to higher dimensional problem spaces. Toggle PRM is a simple coordinated method for mapping both $\mathbb{C}_{free}$ and $\mathbb{C}_{obst}$, with the added theoretical benefit of performing well for narrow passage problems. Additionally, we showed the generalized algorithm's benefit for a variety of benchmark problems. In the future, further extension of the theory behind the method should be taken to possibly show convergence rates, such as was done with PRMs. Further exploration into heuristics needs to be taken for Toggle PRM, for example, if medial-axis retractions are used in conjunction with the Toggle PRM paradigm.

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