A Proof of Theorem 1

Proof. We prove Theorem 1 by induction. First we define \( \delta_{\text{fail}}(b) : B \mapsto \{0, 1\} \) as an indicator and when \( \delta_{\text{fail}}(b) = 1 \), there are no valid partial conditional plans for belief \( b \) and execution fails.

- Base case \((k = 1)\): Since \( \gamma_1^p = (b, a, O_1^p, \emptyset) \) is valid, for every covered observation \( o \in O_1^p \), \( b' = T_B(b, a, o) \in \text{Dest} \) is the terminal goal belief and thus \( \delta_{\text{fail}}(b') = 0 \). Therefore,

\[
p_{\text{fail}}(\gamma_1^p) = \sum_{o \in O - O_1^p} \Pr(o|b, a) \delta_{\text{fail}}(b') \\
\leq \sum_{o \in O - O_1^p} \Pr(o|b, a) = p_{\text{replan}}(\gamma_1^p)
\]

since \( \delta_{\text{fail}}(b') \leq 1 \) where \( b' = T_B(b, a, o) \) is the successor belief for the uncovered observation \( o \in O - O_1^p \).

- Inductive case \((k > 1)\): Since \( \gamma_k^p = (b, a, O_k^p, \nu_k^p) \) is valid, for every covered observation \( o \in O_k^p \), the corresponding \((k - 1)\)-step partial conditional plan \( \nu_k^p(o) \) is also valid. Assume \( p_{\text{fail}}(\nu_k^p(o)) \leq p_{\text{replan}}(\nu_k^p(o)) \), then

\[
p_{\text{fail}}(\gamma_k^p) = \sum_{o \in O_k^p} \Pr(o|b, a) p_{\text{fail}}(\nu_k^p(o)) + \sum_{o \in O - O_k^p} \Pr(o|b, a) \delta_{\text{fail}}(b') \\
\leq \sum_{o \in O_k^p} \Pr(o|b, a) p_{\text{replan}}(\nu_k^p(o)) + \sum_{o \in O - O_k^p} \Pr(o|b, a) \\
= p_{\text{replan}}(\gamma_k^p)
\]

since \( \delta_{\text{fail}}(b') \leq 1 \) where \( b' = T_B(b, a, o) \) is the successor belief for the uncovered observation \( o \in O - O_k^p \).

Therefore, For any \( k \)-step valid partial conditional plan \( \gamma_k^p = (b, a, O_k^p, \nu_k^p) \),

\[
p_{\text{fail}}(\gamma_k^p) \leq p_{\text{replan}}(\gamma_k^p).
\]

\( \square \)