Problem 41 from Open Problems Project
http://maven.smith.edu/~orourke/TOPP/
Problem

- Sorting $X + Y$
  where $X + Y = \{x + y \mid x \in X, y \in Y\}$ (Minkowski sum)

Ex: $X = \{1, 2, 3\}$, $Y = \{4, 5, 6\}$

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
<td>$1 + 4$</td>
<td>$2 + 4$</td>
<td>$3 + 4$</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>$1 + 5$</td>
<td>$2 + 5$</td>
<td>$3 + 5$</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>$1 + 6$</td>
<td>$2 + 6$</td>
<td>$3 + 6$</td>
</tr>
</tbody>
</table>

$\{5, 6, 6, 7, 7, 7, 8, 8, 9\}$
Origin

- Michael Fredman, 1976
- Algorithm found by William Steiger and Ileana Streinu in 1995 with $O(n^2\log n)$ running time with $O(n^2)$ comparisons
- Question: is it $o(n^2\log n)$?
Progress

- Antonio Herrera, 1996
  - Presented problems that are at least as hard as sorting $X + Y$
  - Tracing the intersections of a ray
  - Enumerating distances (K shortest distances)
    - $O(n \log n + k \log n)$
Progress

- Timothy Chan, 2006
  - Necklace alignment ($L_1, L_2, L_\infty$)
Progress

- Timothy Chan, 2006
  - Convolution (max, min, range)
Summary

- **Status**
  - Open
  - $o(n^2 \log n)$

- **Importance**
  - 3SUM ($a + b + c = 0$)
  - Topological Sorting
  - Sperm Sorting