CPSC 411, Fall 2008

HW7 Solutions

Ex. 34.1-4 The running time of the algorithm is $O(n \cdot W)$, where n is the number of items and W is the maximum weight the thief can steal. Assuming numeric parameters to the problem are written in binary, it takes $\log W$ bits to represent W. So the size of the part of the input representing W is $k = \log W$. But the running time depends on $W = 2^k$, which is not polynomial in k (size of the input) in general.

Ex. 34.2-1 Show GRAPH-ISOMORPHISM is in NP by describing a poly time algorithm to verify a candidate solution. A candidate solution is a mapping f from V_1 (nodes of G_1) to V_2 (nodes of V_2). To verify, check that the mapping f is one-to-one and onto (takes time $O(V_1)$). Then check, for each pair of nodes u and v in V_1, that (u,v) is an edge of G_1 if and only if $(f(u), f(v))$ is an edge of G_2 (takes time $O(V_1^2)$).

Ex. 34.5-1 Show that the subgraph isomorphism problem (SI) is NP-complete.

$SI \in NP$: A candidate solution, given input G_1 and G_2, is a subset S of the nodes of G_2 and a mapping f from the nodes of G_1 to S. Verify as in previous exercise.

Known NPC problem ↙ ↙ unknown NPC problem

CLIQUE $\leq_p$ SI : Given any CLIQUE input G and K, construct in poly-time this SI input : G_K and G, where G is the original CLIQUE input and G_K is the clique graph with K nodes. Check that G has a clique of size K if and only if G_K is isomorphic to a subgraph of G (i.e., if and only if G contains a clique of size K). The point is the CLIQUE is a special case of SI.

Prob 34-1

a) Independent set (IS) decision problem:
Given a graph G and an integer K, does G have an independent set of size at least K?
Show IS is NP-complete.

i) IS $\in$ NP: Given a candidate solution, which is a subset S of the nodes of the input graph G, check in polynomial time if $|S| \geq K$ and if there is no edge between any pair of nodes in S.

ii) CLIQUE $\leq_p$ IS : Given an arbitrary CLIQUE input (G, K), construct in polynomial time an IS input $(\overline{G}, K)$, where $\overline{G}$ is the complement graph of G. Since a set of nodes C is a clique in G if and only if C is an independent set in $\overline{G}$, G has a clique of size K if and only if $\overline{G}$ has an independent set of size K.

c) Efficient alg. to solve IS when each vertex in G has degree 2.

Then G must consist of one or more (simple) cycles. For each cycle K, number the nodes in order around the cycle $v_1^K, v_2^K, v_3^K, \ldots$

Choose the even-indexed nodes to be in the independent set: